Section 3.8

Multiply $(x+2)(x+2)$

$x^2+2x+2x+4$

x^2+4x+4

(perfect square trinomial)

Multiply $(x+2)(x-2)$

$x^2-2x+2x-4$

x^2-4

* Difference of squares
* only time (binomial)(binomial) = binomial

Factoring Difference of Squares

1. x^2-25

$(x+5)(x-5)$

$\sqrt{x^2}=x$

$\sqrt{25}=5$

2. x^2-144

$(x+12)(x-12)$

3. $4x^2-36$

$(2x-6)(2x+6)$

4. $9x^2-1$

$(3x-1)(3x+1)$

5. $9x^2+36$

GCF factoring

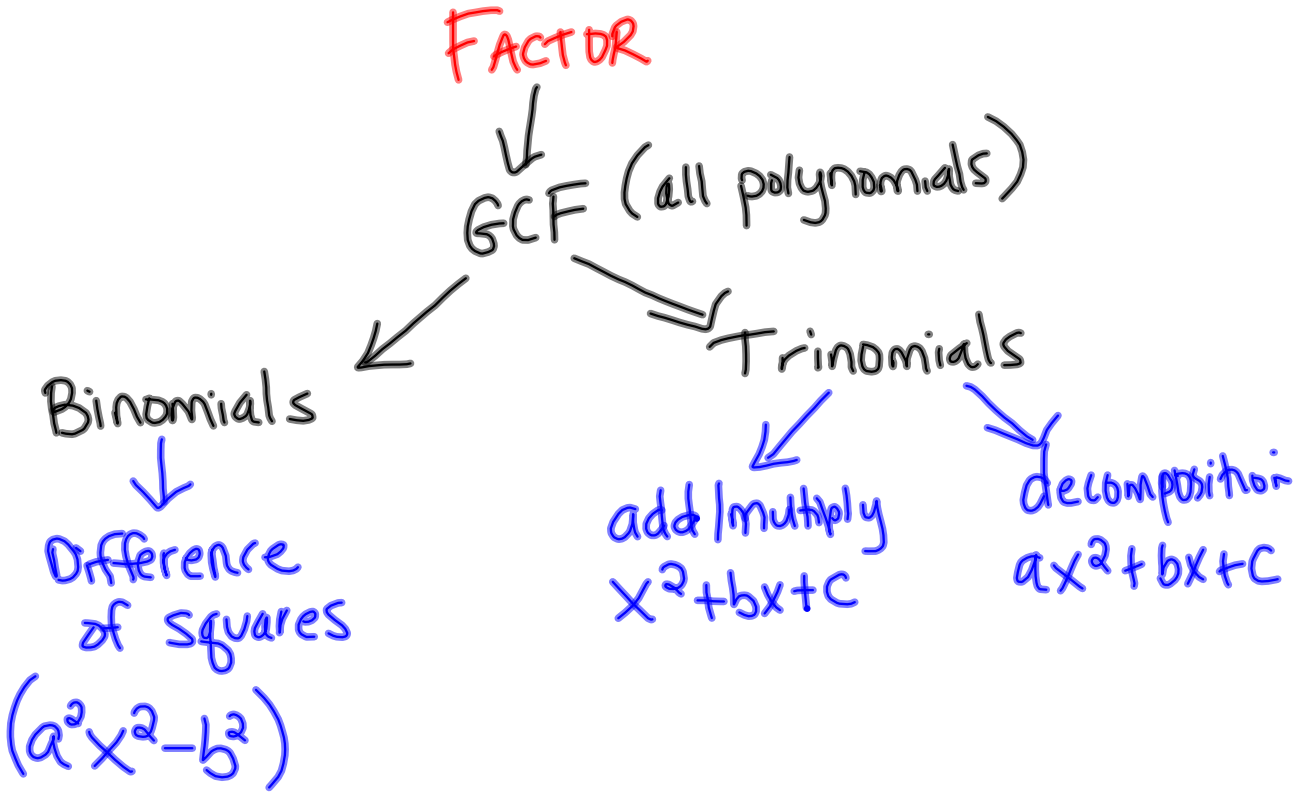
$9(x^2+4)$

6. $8x^2-2$

GCF

$2(4x^2-1)$ ← difference of squares

$2(2x+1)(2x-1)$


FACTOR
GCF (all polynomials)
Binomials
Difference of squares
$(a^2x^2-b^2)$
Trinomials
add/multiply
x^2+bx+c
decomposition
ax^2+bx+c

B. DIFFERENCE OF SQUARES

As we have just seen, perfect square trinomials are formed when a binomial is multiplied by itself. That is,

$$(x + c)(x + c) = x^2 + cx + cx + c^2$$
$$= x^2 + 2cx + c^2$$

and

$$(ax + c)(ax + c) = (ax)^2 + acx + acx + c^2$$
$$= (ax)^2 + (2ac)x + (c)^2$$

There is another type of factoring known as the Difference of Squares that results when we multiply two binomials with *opposite middle signs* of the form $(x + a)(x - a)$. Using the FOIL method we get:

$$(x + a)(x - a) = x^2 - ax + ax - a^2 = x^2 - a^2$$

For example: $(x + 5)(x - 5) = x^2 - 5x + 5x - 5^2 = x^2 - 25$

To factor a difference of squares, you can think of taking the square root of each term.

For example: $4x^2 - 49 = (\sqrt{4x^2} + \sqrt{49})(\sqrt{4x^2} - \sqrt{49}) = (2x + 7)(2x - 7)$

EXAMPLE 12

Factor each of the following.

a. $x^2 - 36$ b. $9x^2 - 16y^2$ c. $8x - 32x^3$

3x 4y

Solution

a. $(x + 6)(x - 6)$

b. $(3x + 4y)(3x - 4y)$

c. $8x(1 - 4x^2)$
$= 8x(1 + 2x)(1 - 2x)$

TRY THESE!

Try the following exercises to see how well you recognize the patterns.

1. Expand each binomial:

a. $(x + 2)(x + 2)$ b. $(x - 3)(x + 3)$ c. $(x + 6)(x - 6)$

d. $(x - 7)(x + 7)$ e. $(x + 9)(x - 9)$ f. $(x - 10)(x + 10)$

$x^2 + 7x - 7x - 49$

2. Complete the following by determining the value that would make each expression a difference of squares:

a. $x^2 - \square$ $= (x-5)(x+5)$

b. $x^2 - \square$ $= (x-11)(x+11)$

c. $x^2 - \square$ $= (x-5)(x+5)$

d. $x^2 - \square$ $= (x-y)(x+y)$

e. $9x^2 - \square$ $= (3x-2y)(3x+2y)$

f. $49x^2 - \square$ $= (7x-6y)(7x+6y)$

3. Factor each difference of squares:

a. $x^2 - 1$ $= (\quad)(\quad)$

b. $x^2 - 16$ $= (\quad)(\quad)$

c. $x^2 - 49$ $= (\quad)(\quad)$

d. $x^2 - 64y^2$ $= (\quad)(\quad)$

e. $x^2 - \frac{25}{4}$ $= (\quad)(\quad)$

f. $x^2 - 1.44y^2$ $= (\quad)(\quad)$

PRACTICE

44. Factor completely.

a. $x^2 - 9$ $(x+3)(x-3)$

b. $a^2 - 36$

c. $p^2 - 100$

d. $4x^2 - 1$

e. $y^2 - 25$

f. $t^2 - 49$

g. $9h^2 - 64$

h. $100 - 81d^2$

i. $36m^2 - 121$

45. Factor completely.

e. $3k^2 - 75$

f. $25x^2 - 9y^2$

g. $x^2 - 16y^2$

h. $63 - 7t^2$

i. $7x^2y^2 - 28x^2z^2$ GCF $7x^2$ $7x^2(y^2 - 4z^2)$ $7x^2(y-2z)(y+2z)$

j. $3p^2 - 27q^2$

k. $81m^2 - 9n^2$

l. ~~$16a^2 - 64b^2$~~

m. $x^4 - 81$