Key

Eastern School District

# Mathematics 2200
# Common Mathematics Assessment

## June 12, 2013

Name: _____

Mathematics Teacher: _____

27 Selected Response — 27 marks
11 Constructed Response — 40 marks

**FINAL** — **67 Marks**

**TIME: 2 HOURS**

**NOTE**

Diagrams are not necessarily drawn to scale.

**FORMULAE**

$t_n = t_1 + (n-1)d \quad , n \in N$

$t_n = t_1 r^{n-1} \quad , n \in N$

$S_n = \frac{n}{2}(t_1 + t_n)$

$S_n = \frac{t_1(r^n - 1)}{r - 1}$

$S = \frac{t_1}{1 - r}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$a^2 = b^2 + c^2 - 2cb\cos A$

$\cos A = \frac{b^2 + c^2 - a^2}{2cb}$

**Selected Response:** Choose the appropriate response on the answer sheet or SCANTRON.

1. What is $S_{16}$ for $t_n = 3(2)^{n-1}$?

   (A) 98 301
   (B) 98 304
   (C) 196 605
   (D) 786 456

$S_{16} = \frac{3(2^{16}-1)}{2-1} = 196605$

2. What is the general term for $\{-40, -43, -46, -49, -52, \ldots\}$?

-37 (-3, -3)

   (A) $t_n = -3n - 43$
   (B) $t_n = -3n - 37$
   (C) $t_n = 3n - 43$
   (D) $t_n = 3n - 37$

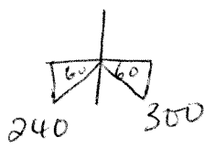
3. The sum of an infinite geometric sequence is 32. If the common ratio is $\frac{3}{4}$, what is the value of $t_1$?

   (A) 8
   (B) 24
   (C) 56
   (D) 128

$32 = \frac{t_1}{1-\frac{3}{4}}$ $\quad 32\left(\frac{1}{4}\right) = t_1$ $\quad 8 = t_1$

4. Solve for $\theta$, where $0° \leq \theta < 360°$: $\sin\theta = -\frac{\sqrt{3}}{2}$

   (A) $\theta = 60°$ and $\theta = 300°$
   (B) $\theta = 150°$ and $\theta = 210°$
   (C) $\theta = 210°$ and $\theta = 330°$
   (D) $\theta = 240°$ and $\theta = 300°$




5. How many unique triangles can be constructed given that $\angle A = 22°$, a = 12, and $b = 22$?

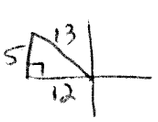
   (A) 0
   (B) 1
   (C) 2
   (D) 3

$\frac{\sin 22°}{12} = \frac{\sin B}{22}$

$B = 43°$ or $137°$ ∴ 2 Δ's.

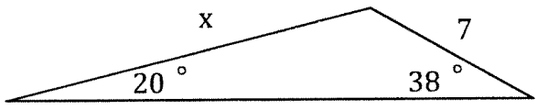
6. An angle, $\theta$, is in standard position with its terminal arm in quadrant II. If $\sin\theta = \frac{5}{13}$, what is the value of $\tan\theta$?

   (A) $-\frac{12}{5}$
   (B) $-\frac{5}{12}$
   (C) $\frac{5}{12}$
   (D) $\frac{12}{5}$




$\tan\theta = \frac{y}{x} = \frac{5}{-12}$

7. What is the length of $x$?

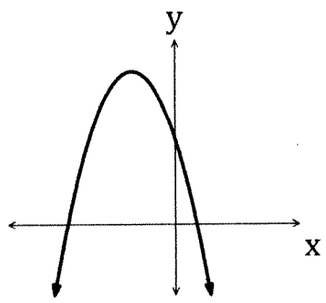



$\frac{\sin 20^\circ}{7} = \frac{\sin 38^\circ}{x}$

$x = 12.6$

(A) 3.9
(B) 5.9
(C) 9.6
(D) 12.6

8. Which equation is represented by the graph?




(A) $y = -3(x-2)^2 + 27$
(B) $y = -3(x+2)^2 - 27$
(C) $y = -3(x+2)^2 + 27$
(D) $y = -3(x-2)^2 - 27$

9. What is the range of $y = 2x^2 + 4x - 7$?

(A) $\{y|y \geq -13, y \in \mathbb{R}\}$
(B) $\{y|y \geq -9, y \in \mathbb{R}\}$
(C) $\{y|y \geq -7, y \in \mathbb{R}\}$
(D) $\{y|y \geq -1, y \in \mathbb{R}\}$

$Vx = \frac{-4}{2(2)} = -1$

$Vy = 2(-1)^2 + 4(-1) - 7 = 2 - 4 - 7 = -9$

10. Write $y = x^2 - 8x + 3$ in the form $y = a(x-p)^2 + q$.

(A) $y = (x-4)^2 - 13$
(B) $y = (x-4)^2 - 5$
(C) $y = (x-4)^2 + 3$.
(D) $y = (x-4)^2 + 19$.

$y = (x^2 - 8x + 16) + 3 - 16$

$y = (x-4)^2 - 13$

11. What value(s) of $k$ will make $x^2 - 6x + k$ a perfect square trinomial?

(A) $\pm 9$
(B) $\pm 36$
(C) 9
(D) 36

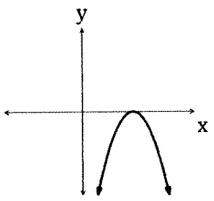
$x^2 - 6x + 9$

12. Which represents a quadratic function in the form $y = a(x - p)^2 + q$, where $a < 0, p < 0, and\ q = 0$?

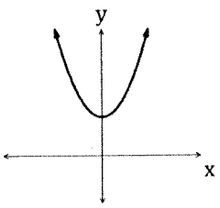
$y = (x - -p)^2 + 0$

$y = (x+p)^2$

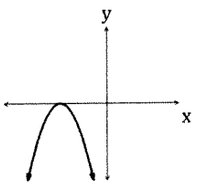
(A)



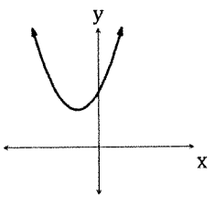

(B)




(C)



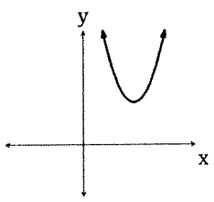

(D)



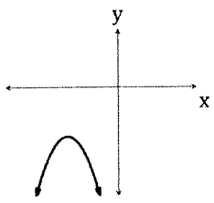

13. Which is a quadratic function with a positive discriminant?

2 diff x-int.

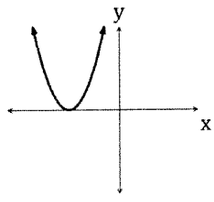
(A)



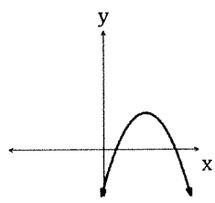

(B)




(C)




(D)



14. Which quadratic function has zeros of $\frac{3}{2}$ and $-5$ ?

(A) $f(x) = 2x^2 - 7x - 15$
(B) $f(x) = 2x^2 - 13x + 15$
(C) $f(x) = 2x^2 + 7x - 15$
(D) $f(x) = 2x^2 + 13x - 15$

$(2x-3)(x+5)$
$2x^2 + 10x - 3x - 15$
$2x^2 + 7x - 15$

15. What is the simplest form of $\sqrt[3]{54x^5y^6z^8}$ ?

(A) $2xy^2z^2\sqrt[3]{3x^2z^2}$
(B) $3x^2y^3z^4\sqrt[3]{6x}$
(C) $3xy^3z^2\sqrt[3]{2x^2z^2}$
(D) $3xy^2z^2\sqrt[3]{2x^2z^2}$

$\sqrt[3]{27 \cdot 2 \cdot x^3 \cdot x^2 \cdot y^6 \cdot z^6 \cdot z^2}$
$= 3xy^2z^2\sqrt[3]{2x^2z^2}$

16. Simplify completely: $\frac{2}{7}\sqrt{98} - \frac{3}{2}\sqrt{8} + \frac{4}{5}\sqrt{50}$

(A) $\sqrt{2}$
(B) $3\sqrt{2}$
(C) $9\sqrt{2}$
(D) $28\sqrt{2}$

$\frac{2}{7}\sqrt{49 \cdot 2} - \frac{3}{2}\sqrt{4 \cdot 2} + \frac{4}{5}\sqrt{25 \cdot 2}$
$2\sqrt{2} - 3\sqrt{2} + 4\sqrt{2} = 3\sqrt{2}$

17. Simplify completely: $\frac{6\sqrt{12x^{16}}}{2\sqrt{18x^9}}$

(A) $x\sqrt{16}$
(B) $x^3\sqrt{6x}$
(C) $\frac{3}{2}x^3\sqrt{3x}$
(D) $\frac{3}{2}x^3\sqrt{6x}$

$\frac{3\sqrt{4 \cdot 3x^{16}}}{\sqrt{9 \cdot 2x^8 \cdot x}} = \frac{6x^8\sqrt{3} \cdot \sqrt{2x}}{3x^4\sqrt{2x}\sqrt{2x}}$
$= \frac{2x^4\sqrt{6x}}{2x}$ $x^3\sqrt{6x}$

18. An incorrect simplification is provided. In which step does the **first** error occur?

*Simplify:* $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}$

*Solution:*

*Step 1* $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}} \cdot \frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}}$
*Step 2* $\frac{\sqrt{9}+\sqrt{25}}{\sqrt{9}-\sqrt{25}}$
*Step 3* $\frac{3+5}{3-5}$
*Step 4* $4$

(A) 1
(B) 2
(C) 3
(D) 4

19. Simplify: $\dfrac{2+\frac{1}{x}}{4x-\frac{1}{x}}$

$\dfrac{\frac{2x+1}{x}}{\frac{4x^2-1}{x}} = \dfrac{2x+1}{x} \cdot \dfrac{x}{(2x+1)(2x-1)} = \dfrac{1}{2x-1}$

(A) $\dfrac{-1}{2x}$

(B) $\dfrac{-2x+1}{2x}$

(C) $\dfrac{1}{2x-1}$

(D) $\dfrac{2x+1}{4x-1}$

20. Simplify: $\dfrac{2-x}{x^2-16} \div \dfrac{x^2+x-6}{x^2+7x+12}$

$\dfrac{-(x-2)}{(x+4)(x-4)} \times \dfrac{(x+4)(x+3)}{(x+3)(x-2)} = -\dfrac{1}{x-4}$

(A) $\dfrac{-1}{x-4}$

(B) $\dfrac{1}{x-4}$

(C) $\dfrac{(2-x)(x-2)}{(x-4)(x+4)^2}$

(D) $\dfrac{(2-x)}{(x-4)(x-2)}$

21. Simplify: $\frac{1}{x}+\frac{2}{x+3}$

$\dfrac{1(x+3)+2(x)}{x(x+3)} = \dfrac{3x+3}{x(x+3)} = \dfrac{3(x+1)}{x(x+3)}$

(A) $\dfrac{3}{x}$

(B) $\dfrac{2x}{x+3}$

(C) $\dfrac{3}{2x+3}$

(D) $\dfrac{3(x+1)}{x(x+3)}$

22. What is the domain of $y = |x-4|$?

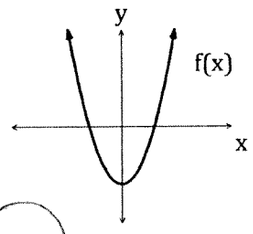
(A) $\{x|x \in \mathbb{R}\}$

(B) $\{x|x \geq 4, x \in \mathbb{R}\}$

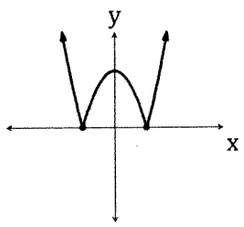
(C) $\{x|x \leq 4, x \in \mathbb{R}\}$

(D) $\{x|x = 4, x \in \mathbb{R}\}$

23. Given $y = f(x)$, which represents $y = |f(x)|$?




(A)

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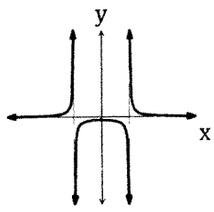
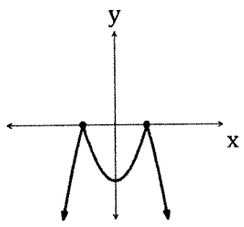
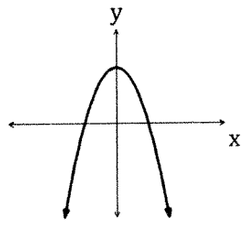



(C)




(D)




24. Which system of equations could be used to solve the given problem:

*Two numbers differ by 14. When the smaller is subtracted from the square of the larger, the result is 394. What are the numbers?*

(A) $\begin{cases} x - y = 14 \\ x^2 - y = 394 \end{cases}$

(B) $\begin{cases} x - y = 14 \\ y^2 - x = 394 \end{cases}$

(C) $\begin{cases} x - 14 = y \\ y - x^2 = 394 \end{cases}$

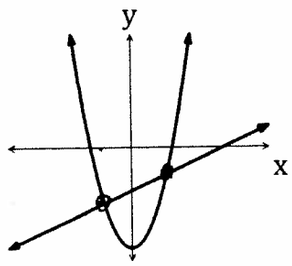
(D) $\begin{cases} x - 14 = y \\ x - y^2 = 394 \end{cases}$

L S

$x - y = 14$

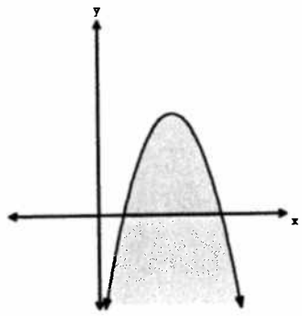
$x^2 - y = 394$

25. What are the solutions for the system shown?




(A) $(-3, 0)$ *and* $(3, 0)$
(B) $(-2, -4)$ *and* $(2, -2)$
(C) $(0, -3)$ *and* $(0, -6)$
(D) $(0, -3)$ *and* $(6, 0)$

26. Which inequality is graphed?




(A) $y < -(x-3)^2 + 4$
(B) $y \geq -(x-3)^2 + 4$
(C) $y \leq -(x-3)^2 + 4$
(D) $y \geq -(x-3)^2 + 4$

27. Which is a solution to $x - 4y > 8$?

(A) $(-2, 0)$
(B) $(0, -2)$
(C) $(1, -4)$
(D) $(4, -1)$

**Constructed Response:**
Answers to be written on this paper in the space provided. Show all workings.

28. A theatre has 11 seats in the first row. Each row has 2 more seats than the previous row, and there are 12 rows in total. Algebraically determine the total capacity of the theatre. [3 marks]

$11, 13, 15, \ldots$

$t_n = t_1 + (n-1)d$ ✓

$t_{12} = 11 + (12-1)2$ ✓

$= 11 + 22$

$t_{12} = 33$ ✓

$S_n = \frac{n}{2}(t_1 + t_n)$ ✓

$S_{12} = \frac{12}{2}(11 + 33)$ ✓

$= 6(44)$

$= 264$ ✓

(1½)

(1½)

Total capacity is 264.

29. The first three terms of a geometric sequence are $\{x-1, 2x, 3x+9, \ldots\}$. Algebraically determine the value of $x$. [3 marks]

$\frac{3x+9}{2x} = \frac{2x}{x-1}$ (1)

$(2x)(2x) = (3x+9)(x-1)$

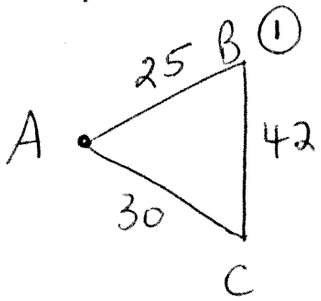
$4x^2 = 3x^2 + 6x - 9$

(1) $x^2 - 6x + 9 = 0$

(½) $(x-3)(x-3) = 0$

$x = 3$ (½)

30. Two sailboats leave the same point at the same time, travelling in different directions. One boat travels at 5 km/h and the other travels at 6 km/h. If the boats are 42 km apart after 5 hours, find the measure of the angle between the paths of the boats. [4 marks]




(1)

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (1)

$\cos A = \frac{30^2 + 25^2 - 42^2}{2(30)(25)}$

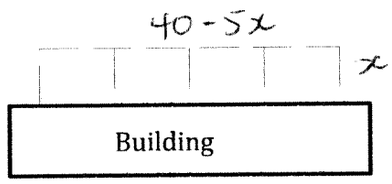
$\cos A = \frac{-239}{1500}$ (1)

$\cos A = -0.1593$

$A = \cos^{-1}(0.1593) = 99°$ (1)

The angle between the paths of the boats is 99°.

31. Four equivalent rectangular areas are enclosed along the side of a building as shown. If 40 m of fencing is used, algebraically determine the dimensions that will maximize the enclosed area. [4 marks]




$x$ = width (1)

$40 - 5x$ = length

Area $= x(40-5x)$ (1)

$A = 40x - 5x^2$

(1) max. at $V_x = \frac{-b}{2a} = \frac{-40}{2(-5)} = 4$

$\therefore$ width $= 4$

length $= 40 - 5(4) = 20$ (1)

32. Algebraically determine the **exact** roots, in simplest form, for: [4 marks]

$5x(5x + 4) = 13$

(1) $25x^2 + 20x - 13 = 0$

(1) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-20 \pm \sqrt{20^2 - 4(25)(-13)}}{2(25)}$

($\frac{1}{2}$) $x = \frac{-20 \pm \sqrt{400 + 1300}}{50}$

($\frac{1}{2}$) $x = \frac{-20 \pm \sqrt{1700}}{50}$

($\frac{1}{2}$) $x = \frac{-20 \pm 10\sqrt{17}}{50}$

($\frac{1}{2}$) $x = \frac{-2 \pm \sqrt{17}}{5}$

33. State **restrictions** on the variable and **solve**: $n - \sqrt{3-n} = -9$ [4 marks]

Restrictions ($\frac{1}{2}$) $3 - n \geq 0$

$3 \geq n$ or $n \leq 3$.

($\frac{1}{2}$) $n - \sqrt{3-n} = -9$

$n + 9 = \sqrt{3-n}$

$(n+9)^2 = (\sqrt{3-n})^2$

$n^2 + 18n + 81 = 3 - n$

(1) $n^2 + 19n + 78 = 0$

($\frac{1}{2}$) $(n+6)(n+13) = 0$

$n = -6$ or $n = -13$ ($\frac{1}{2}$)

Check:

$n = -13$

$-13 - \sqrt{3 - -13}$

$\Rightarrow -13 - \sqrt{16}$

$\Rightarrow -13 - 4$

$\Rightarrow -17$

$\neq -9$ reject ($\frac{1}{2}$)

$n = -6$

$-6 - \sqrt{3 - -6}$

$\Rightarrow -6 - \sqrt{9}$

$\Rightarrow -6 - 3$

$\Rightarrow -9$

$= -9$ ($\frac{1}{2}$)

$\therefore n = -6$

34. Identify all non-permissible values and **solve**: $\frac{2m}{m-1} + \frac{m-5}{m^2-1} = 1$ [4 marks]

n.p. values: $m \neq \pm 1$ (½)

$(m+1)(m-1)$ (½)

(½) $(m+1)(m-1)\left[\frac{2m}{m-1}\right] + (m+1)(m-1)\left[\frac{m-5}{(m+1)(m-1)}\right] = (m+1)(m-1)[1]$

(½) $2m(m+1) + m - 5 = (m+1)(m-1)$

(½) $2m^2 + 2m + m - 5 = m^2 - 1$

(½) $m^2 + 3m - 4 = 0$

(½) $(m+4)(m-1) = 0$

$m = -4$ $m = 1$ Reject (½)

Solution (½)

35. Algebraically determine the invariant points, equations of asymptotes, and x- and y-intercepts for the functions $f(x) = 2x - 6$ and $y = \frac{1}{f(x)}$. Sketch both graphs on the same set of axes. [4 marks] (1)

$y = \frac{1}{2x-6}$ V.A. $x = 3$ H.A. $y = 0$

$y = 2x - 6$

x-int:

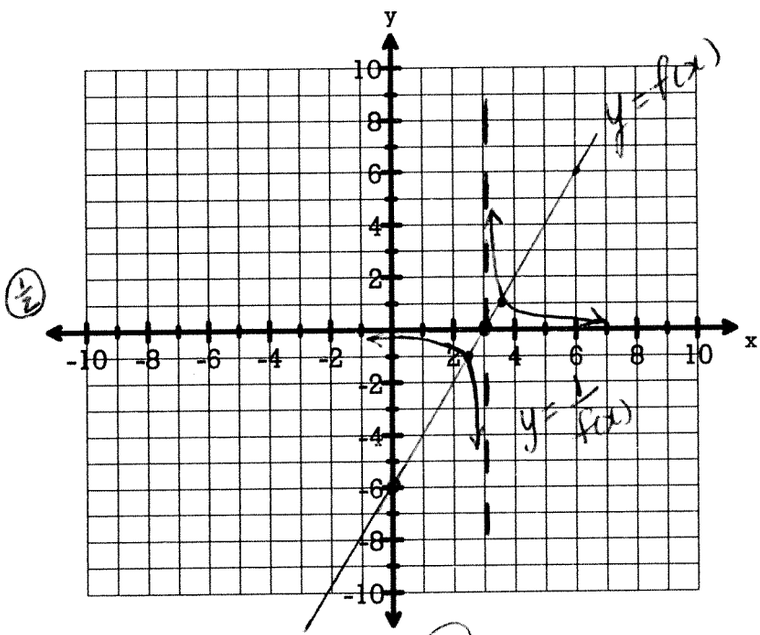
$0 = 2x - 6$

$6 = 2x$

$3 = x$

y-int:

$y = -6$




(½) (1)

Inv. Pts.

$2x - 6 = +1$ (1)

$2x = 7$

$x = \frac{7}{2}$ $(\frac{7}{2}, 1)$

or

$2x - 6 = -1$

$2x = 5$

$x = \frac{5}{2}$ $(\frac{5}{2}, -1)$

x-int: none

(½) y-int: $\frac{1}{2(0)-6} = -\frac{1}{6}$

36. Solve algebraically: $|x(x-3)| = x$ [4 marks]

$(\frac{1}{2})\ +x(x-3) = x$ OR $(\frac{1}{2})\ -x(x-3) = x$

$x^2-3x = x$ $\quad -x^2+3x = x$

$(\frac{1}{2})\ x^2-4x = 0$ $\quad -x^2+2x = 0$

$x(x-4) = 0$ $\quad (\frac{1}{2})\ x^2-2x = 0$

$(\frac{1}{2})\ x = 0, x = 4$ $\quad x(x-2) = 0$

$(\frac{1}{2})\ x = 0, x = 2$

Solution: $\{0, 2, 4\}$

(1) Check: $x = 0$: LS $= |0(0-3)| = |0| = 0 =$ RS ✓

$x = 4$: LS $= |4(4-3)| = |4| = 4 =$ RS ✓

$x = 2$: LS $= |2(2-3)| = |-2| = 2 =$ RS ✓

37. Algebraically determine the points of intersection of $y = 3 - x^2$ and $y = 2x^2 - 5x + 1$. [3 marks]

$(\frac{1}{2})\ 3-x^2 = 2x^2-5x+1$

$0 = 3x^2-5x-2$

$(\frac{1}{2})\ 0 = (3x+1)(x-2)$

$3x+1 = 0$, $x = 2$ $(\frac{1}{2})$

$x = -\frac{1}{3}$

$(\frac{1}{2})$

$x = -\frac{1}{3}$: $y = 3-(-\frac{1}{3})^2$, $y = 3-\frac{1}{9}$, $y = \frac{26}{9}$, $(-\frac{1}{3}, \frac{26}{9})$ $(\frac{1}{2})$

$x = 2$: $y = 3-2^2$, $y = 3-4$, $y = -1$, $(2, -1)$ $(\frac{1}{2})$

points of int.

38. A toy rocket is launched from the roof of a house. Its height, h, in metres above the ground is given by $h(t) = -5t^2 + 40t + 10$, where t is time in seconds. Algebraically determine when the rocket has a height of at least 45 m. [3 marks]

$(\frac{1}{2})\ -5t^2+40t+10 \geq 45$

$-5t^2+40t-35 \geq 0$

$-5(t^2-8t+7) \geq 0$

$(\frac{1}{2})\ t^2-8t+7 \leq 0$ *

$(\frac{1}{2})\ (t-7)(t-1) \leq 0$

$(\frac{1}{2})\ t = 7 \quad t = 1$

(+) | (−) | (+) $(\frac{1}{2})$

1 ~ 7

$t \in [1, 7]$ $(\frac{1}{2})$

Rocket height is at least 45m between 1s and 7s, inclusive.

Mathematics 2200 Common Assessment – June 2013
Answer Sheet

Name: _____

Mathematics Teacher:_____

| | | | | |
|---|---|---|---|---|
| 1. | A | B | **C** | D |
| 2. | A | **B** | C | D |
| 3. | **A** | B | C | D |
| 4. | A | B | C | **D** |
| 5. | A | B | **C** | D |
| 6. | A | **B** | C | D |
| 7. | A | B | C | **D** |
| 8. | A | B | **C** | D |
| 9. | A | **B** | C | D |
| 10. | **A** | B | C | D |
| 11. | A | B | **C** | D |
| 12. | A | B | **C** | D |
| 13. | A | B | C | **D** |
| 14. | A | B | **C** | D |
| 15. | A | B | C | **D** |
| 16. | A | **B** | C | D |
| 17. | A | **B** | C | D |
| 18. | A | **B** | C | D |
| 19. | A | B | **C** | D |
| 20. | **A** | B | C | D |
| 21. | A | B | C | **D** |
| 22. | **A** | B | C | D |
| 23. | **A** | B | C | D |
| 24. | **A** | B | C | D |
| 25. | A | **B** | C | D |
| 26. | A | B | **C** | D |
| 27. | A | B | **C** | D |