

## Section 6.2 continued

## Double-Angle Identities

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

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Write as a single trig expression

$$\cos^2 \frac{\pi}{3} - \sin^2 \frac{\pi}{3} \quad \leftarrow \text{Cos 2A Identity}$$

$$\cos 2\left(\frac{\pi}{3}\right)$$

$$\cos \frac{2\pi}{3}$$

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Determine an identity for  $\cos 2A$  that contains only cosine.

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= \cos^2 A - (1 - \cos^2 A) \end{aligned}$$

$\left. \begin{array}{l} \cos^2 x + \sin^2 x = 1 \\ \sin^2 x = 1 - \cos^2 x \end{array} \right\}$

$$\cos 2A = \cos^2 A - 1 + \cos^2 A$$

$$\boxed{\cos 2A = 2\cos^2 A - 1}$$

↑ Another version of the Identity

Write  $\cos 2A$  identity using only sine.

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= (1 - \sin^2 A) - \sin^2 A \end{aligned}$$

$$\boxed{\cos 2A = 1 - 2\sin^2 A}$$

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Determine the exact value for

a)  $\sin \frac{\pi}{12}$

Special angles

$$\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{2}$$

$$\sin\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$$

$$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \leftarrow \text{Identity}$$

$$\frac{2\pi}{12}, \frac{4\pi}{12}, \frac{3\pi}{12}, \frac{6\pi}{12}$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4}$$

$$\left(\frac{\sqrt{3}}{2}\right) \left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right) \left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$b) \tan 105^\circ$$

$$\tan(45^\circ + 60^\circ)$$

$$\boxed{\text{Identity}}$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{or } \tan(135^\circ - 30^\circ)$$

$$\frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$$

rationalize the denominator

$$\frac{2\sqrt{3} + 4}{-2}$$

$$\boxed{-\sqrt{3} - 2}$$

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#1 b, c, d, e

#2. c, d

4. a, b, c

5 a, b

8 a  $\rightarrow$  e