6.4 Solving Trignometric Equations using Identities

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ex. 1. Solve $\cos 2x + 1 - \cos x = 0$ for $0 \le x < 2\pi$

Identity

$2\cos^2 x - 1 + 1 - \cos x = 0$

$2\cos^2 x - \cos x = 0$

$\cos x (2\cos x - 1) = 0$

$\cos x = 0$: $\frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x = \frac{1}{2}$: $\frac{\pi}{3}, \frac{5\pi}{3}$

ex. 2) $1 - \cos^2 x = 3\sin x - 2$

Identity

$\sin^2 x = 3\sin x - 2$

$\sin^2 x - 3\sin x + 2 = 0$

add $= -3$

mult $= 2$

$-1, -2$

$(\sin x - 1)(\sin x - 2) = 0$

$\sin x = 1$

$x = \frac{\pi}{2}$

$\sin x = 2$

no solution

3) $\cos^2 x = \cot x \sin x \quad 0° \le x < 360°$

$\cos^2 x = \frac{\cos x}{\sin x} \cdot \sin x$

$\cos^2 x = \cos x$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0$: $90°, 270°$

$\cos x = 1$: $0°$

4) $\sin 2x = \sqrt{2}\cos x$ for all x in radians.

Identity

$2\sin x \cos x = \sqrt{2}\cos x$

$2\sin x \cos x - \sqrt{2}\cos x = 0$

$\cos x(2\sin x - \sqrt{2}) = 0$

$\cos x = 0$: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = \frac{\sqrt{2}}{2}$: $x = \frac{\pi}{4} + \frac{3\pi}{4}$

$x = \frac{\pi}{2} + \pi n, n \in I$

$x = \frac{\pi}{4} + 2\pi n, n \in I$

$x = \frac{3\pi}{4} + 2\pi n, n \in I$

ex. 5) $2\sin x = 7 - 3\csc x$

$2\sin x = 7 - \frac{3}{\sin x} \cdot (\sin x)$

$2\sin^2 x = 7\sin x - 3$

$2\sin^2 x - 7\sin x + 3 = 0$

add $\Rightarrow -7$

mut $\Rightarrow 6$

$-1, -6$

$(2\sin^2 x - \sin x)(-6\sin x + 3) = 0$

$\sin x(2\sin x - 1) - 3(2\sin x - 1) = 0$

$(\sin x - 3)(2\sin x - 1) = 0$

$\sin x = 3$ no solution

$\sin x = \frac{1}{2}$

$\frac{\pi}{6} + 2\pi n, n \in I$

$\frac{5\pi}{6} + 2\pi n, n \in I$

Practice

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