# 1.2: Reflections & Stretches

## Reflection:

Reflection of a graph creates a mirror image in a line called the line of reflection. (along x-axis, along y-axis)

→ do not change the shape of the graph.

$y = -f(x)$ → reflection along the x-axis

$y = f(-x)$ → reflection along y-axis.

## Vertical & Horizontal Stretches

↳ stretch factor changes the shape of the graph.

$y = a\,f(x)$ → vertical stretch factor

ex) $y = 2x^2$ → parabola more narrow

$y = \frac{1}{2}x^2$ → parabola wider

Can also be written $\frac{1}{a}y = f(x)$

* Stretch factors are never negative $|a|$

$y = f(bx) \rightarrow$ Horizontal Stretch factor

$\frac{1}{|b|}$ is the stretch factor.

ex) $y = \sin x \rightarrow$ original period $360°$

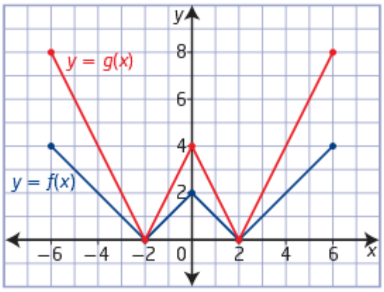
$y = \sin 2(x) \rightarrow$ h.s. $= \frac{1}{2}$ period $180°$

$y = \sin \frac{1}{2}(x) \rightarrow$ h.s $= 2$ period $720°$

Invariant Point: points that stay the same under the transformations.

| $x$ | $y = f(x)$ | $y = g(x) = 2f(x)$ |
|---|---|---|
| −6 | 4 | 8 |
| −2 | 0 | 0 |
| 0 | 2 | 4 |
| 2 | 0 | 0 |
| 6 | 4 | 8 |

The vertical distances of the transformed graph have been changed by a factor of $a$, where $|a| > 1$. The points on the graph of $y = af(x)$ are farther away from the $x$-axis than the corresponding points of the graph of $y = f(x)$.




$y = a f(x)$

vertical stretch factor of 2

Since $a = 2$, the points on the graph of $y = g(x)$ relate to the points on the graph of $y = f(x)$ by the mapping $(x, y) \to (x, 2y)$. Therefore, each point on the graph of $g(x)$ is twice as far from the $x$-axis as the corresponding point on the graph of $f(x)$. The graph of $g(x) = 2f(x)$ is a vertical stretch of the graph of $y = f(x)$ about the $x$-axis by a factor of 2.

The invariant points are $(-2, 0)$ and $(2, 0)$.

What is unique about the invariant points?

V.S affects Range

For $f(x)$, the domain is $\{x \mid -6 \le x \le 6, x \in \mathrm{R}\}$, or $[-6, 6]$, and the range is $\{y \mid 0 \le y \le 4, y \in \mathrm{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \le x \le 6, x \in \mathrm{R}\}$, or $[-6, 6]$, and the range is $\{y \mid 0 \le y \le 8, y \in \mathrm{R}\}$, or $[0, 8]$.

How can you determine the range of the new function, $g(x)$, using the range of $f(x)$ and the parameter $a$?

**Did You Know?**

There are several ways to express the domain and range of a function. For example, you can use words, a number line, set notation, or interval notation.

**Solution**

**a)** Use key points on the graph of $y = f(x)$ to create a table of values.

The image points on the graph of $g(x) = f(2x)$ have the same $y$-coordinates but different $x$-coordinates. Multiply the $x$-coordinates of points on the graph of $y = f(x)$ by $\frac{1}{2}$.

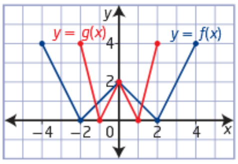
| $x$ | $y = f(x)$ |
|---|---|
| −4 | 4 |
| −2 | 0 |
| 0 | 2 |
| 2 | 0 |
| 4 | 4 |

| $x$ | $y = g(x) = f(2x)$ |
|---|---|
| −2 | 4 |
| −1 | 0 |
| 0 | 2 |
| 1 | 0 |
| 2 | 4 |

$f(2x)$

h.s of $\frac{1}{2}$

$(x, y) \to (\frac{1}{2}x, y)$




The horizontal distances of the transformed graph have been changed by a factor of $\frac{1}{b}$, where $|b| > 1$. The points on the graph of $y = f(bx)$ are closer to the $y$-axis than the corresponding points of the graph of $y = f(x)$.

Since $b = 2$, the points on the graph of $y = g(x)$ relate to the points on the graph of $y = f(x)$ by the mapping $(x, y) \to \left(\frac{1}{2}x, y\right)$. Therefore, each point on the graph of $g(x)$ is one half as far from the $y$-axis as the corresponding point on the graph of $f(x)$. The graph of $g(x) = f(2x)$ is a horizontal stretch about the $y$-axis by a factor of $\frac{1}{2}$ of the graph of $f(x)$.

The invariant point is $(0, 2)$.

For $f(x)$, the domain is $\{x \mid -4 \le x \le 4, x \in R\}$, or $[-4, 4]$, and the range is $\{y \mid 0 \le y \le 4, y \in R\}$.

How can you determine the domain of the new

$\boxed{\text{Sec. 1.1}}$ pg. 12

2a, b

3c, d

4a, 5, 8, 11

$\boxed{\text{Sec. 1.2}}$ pg. 28

2, 3, 5, 6