3.2: The Remainder Theorem

Polynomial Long Division

Recap Numerical Long Division

$$\begin{array}{r} 27 \\ 12\overline{)327} \\ -24 \\ \hline 87 \\ -84 \\ \hline 3 \end{array}$$

3 Remainder

Polynomial Examples

$$\begin{array}{r} x+4 \\ x+3\overline{)x^2+7x+17} \\ -\left(x^2+3x\right) \\ \hline 4x+17 \\ -\left(4x+12\right) \\ \hline 5 \end{array}$$

5 Remainder

Check:

$(x+3)(x+4)+5$

$x^2+7x+12+5$

$x^2+7x+17$

Try:

$$\begin{array}{r} x^2+6x+8 \\ x+1\overline{)x^3+7x^2+14x+8} \\ -\left(x^3+x^2\right) \\ \hline 6x^2+14x \\ -\left(6x^2+6x\right) \\ \hline 8x+8 \\ 8x+8 \\ \hline 0 \end{array}$$

0 Remainder

* Because $(x+1)$ divided perfectly, with no remainder, then $(x+1)$ is a factor.

Hence, $x=-1$ is an x-intercept.

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$$\frac{P(x)}{(x-a)} = Q(x) + \frac{R}{(x-a)}$$

or $P(x) = Q(x)(x-a) + R$

polynomial — quotient — divisor (factor) — remainder

$$\begin{array}{r|l} & Q(x) \\ \hline (x-a) & P(x) \\ & \vdots \\ \hline & R \end{array}$$

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$P(x) = 5x^3 + 10x - 13x^2 - 9$

Divide by $(x-2)$ and express in the form

$$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$$

* $P(x)$ is not in descending order

$P(x) = 5x^3 - 13x^2 + 10x - 9$

$$\begin{array}{r|l} & 5x^2 - 3x + 4 \\ \hline x-2 & \cancel{5x^3} - 13x^2 + 10x - 9 \\ - & \cancel{5x^3} - 10x^2 \\ \hline & \quad -3x^2 + 10x \\ & \quad -3x^2 + 6x \\ \hline & \qquad 4x - 9 \\ & \qquad 4x - 8 \\ \hline & \qquad\quad -1 \; R \end{array}$$

$$\frac{5x^3 - 13x^2 + 10x - 9}{(x-2)} = 5x^2 - 3x + 9 \; \frac{-1}{(x-2)}$$

Homework

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