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8a) $x^3 + 4x^2 - x + K \div (x-1)$

Remainder = 3

$(1)^3 + 4(1)^2 - (1) + K = 3$

$1 + 4 - 1 + K = 3$

$K = 3 - 4$

$K = -1$

Sec. 3.3: The Factor Theorem

The factor theorem states that $x-a$ is a factor of $P(x)$, if and only if $P(a)=0$.

ex. Determine if $x-1$ and $x+2$ are factors of $P(x)=x^3-x^2-5x+2$.

$P(1)=(1)^3-(1)^2-5(1)+2$

$=-3$

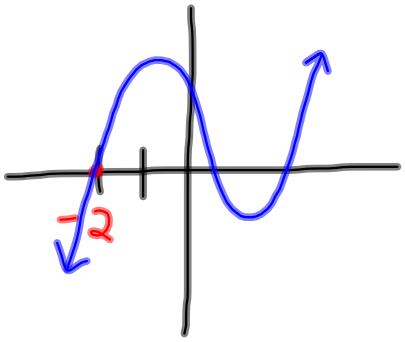
Not a factor

$P(-2)=(-2)^3-(-2)^2-5(-2)+2$

$=0$

$(x+2)$ is a factor

Then -2 is a root.



Integral Zero theorem

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If $x=a$ is an integral zero (root) of polynomial, $P(x)$, with integral coefficients, then 'a' is a factor of the constant term.

example: $2x^3-5x^2-4x+3$

* possible integral roots/zeroes are the factors of 3.

$\pm 1, \pm 3$

Check $P(1)$

$2(1)^3-5(1)^2-4(1)+3$

$2-5-4+3$

$P(1)=-4$ not a root

$P(-1) = 2(-1)^3-5(-1)^2-4(-1)+3$

$= -2-5+4+3$

$= 0$

* -1 is a root

$$\begin{array}{r|rrrr} -1 & 2 & -5 & -4 & 3 \\ & \downarrow & -2 & 7 & -3 \\ \hline & 2 & -7 & 3 & \text{ØR} \end{array}$$

$2x^2 - 7x + 3$

To solve for the other two roots use decomposition or quadratic formula.

$2x^2 - 7x + 3 = 0$

add → −7
mult → 6
−6 & −1

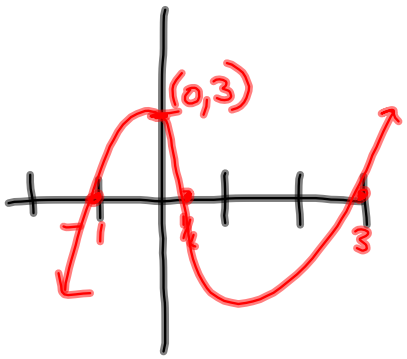
$(2x^2 - 6x)(-x + 3)$

$2x(x-3) - 1(x-3)$

$(2x-1)(x-3) = 0$

$x = \frac{1}{2}$ | $x = 3$

Roots are $-1, \frac{1}{2}$ & 3.



Ex.2 Find all the factors

$x^4 - 5x^3 + 2x^2 + 20x - 24$

possible roots : $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

check: $(2)^4 - 5(2)^3 + 2(2)^2 + 20(2) - 24$

$= 0$

2	1	-5	2	20	-24
	↓	2	-6	-8	24
	1	-3	-4	12	0

$x^3 - 3x^2 - 4x + 12$

possible roots $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

check $P(3) = (3)^3 - 3(3)^2 - 4(3) + 12$

$= 0$

3	1	-3	-4	12
	↓	3	0	-12
	1	0	-4	0

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

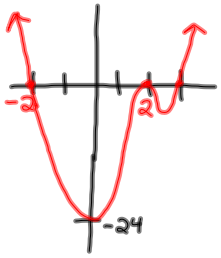
$x = 2 \quad x = -2$

In Factored Form

$(x-2)(x+2)(x-3)(x-2)$

$(x-2)^2(x-3)(x+2)$

double root at 2.



Practice pg. 134

5 a, d, e

Factor fully

5a) $P(x) = x^3 - 6x^2 + 11x - 6$

d) $P(x) = x^4 + 4x^3 - 7x^2 - 34x - 24$

e) $P(x) = K^5 + 3K^4 - 5K^3 - 15K^2 + 4K + 12$