Part 1: Multiple Choice Questions (15 marks)
Complete each multiple choice item and place your answer on the Answer Sheet provided.

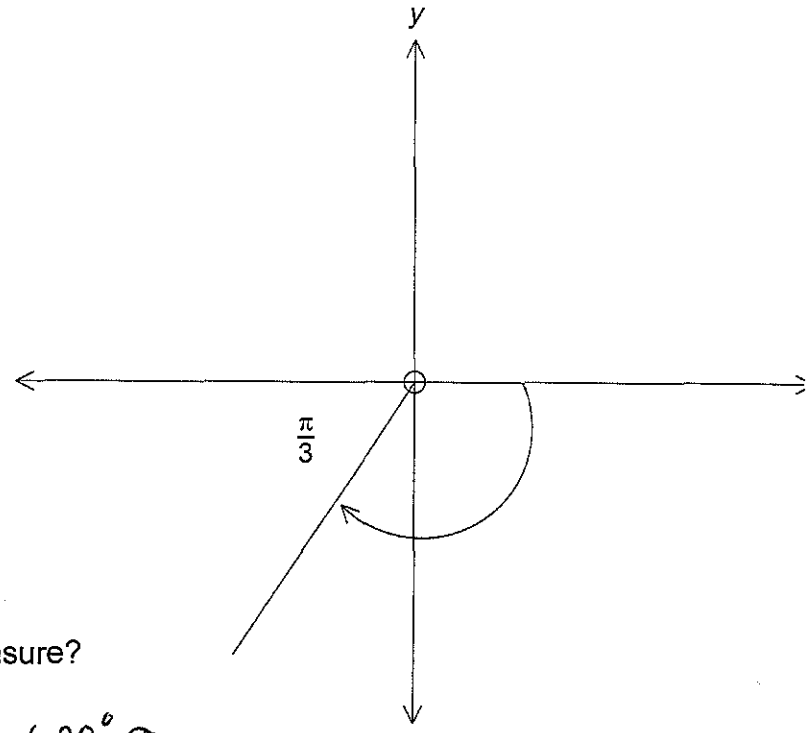
1. What is the measure of this rotation in standard position?

A) $\frac{-5\pi}{6}$

B) $\frac{-2\pi}{3}$

C) $\frac{4\pi}{3}$

D) $\frac{7\pi}{6}$



$\pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$

neg rotation $\therefore -\frac{2\pi}{3}$

2. What is $620°$ written in radian measure?

A) $\frac{11\pi}{9}$

B) $\frac{21\pi}{9}$

C) $\frac{31\pi}{9}$

D) $\frac{41\pi}{9}$

$620° \cdot \frac{\pi}{180°} = \frac{620°}{180°}\pi$

$= \frac{31}{9}\pi$

3. The length of an arc subtended by a central angle of $80°$ is $\frac{20\pi}{9}$. What is the radius?

A) 4

B) 5

C) 6

D) 7

$a = \theta \cdot r$

$80° \cdot \frac{\pi}{180°} = \frac{4\pi}{9}$

$\frac{20\pi}{9} = \frac{4\pi}{9} \cdot r$

$\frac{9}{4\pi} \cdot \frac{20\pi}{9} = r$

$5 = r$

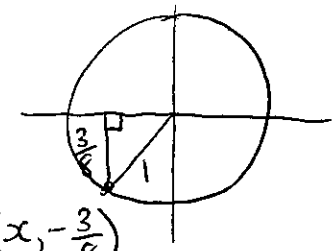
4. What is the equation of a circle with centre at the origin and passing through the point $(4,3)$?

A) $x^2+y^2=\sqrt{5}$
B) $x^2+y^2=5$
C) $x^2+y^2=25$
D) $x^2+y^2=625$

$x^2+y^2=r^2$
$4^2+3^2=r^2$
$16+9=r^2$
$25=r^2$
$\therefore x^2+y^2=25$.

5. Point $P\left(x,-\frac{3}{8}\right)$ lies on the unit circle in Quadrant III. What is the value of x?

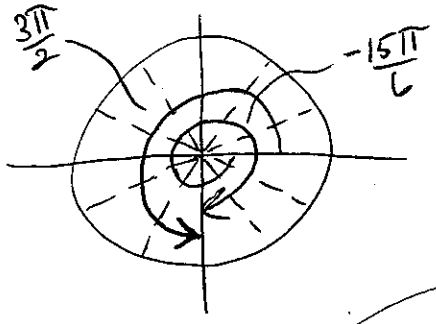
A) $\frac{-55}{64}$
B) $\frac{-\sqrt{55}}{64}$
C) $\frac{-55}{8}$
D) $\frac{-\sqrt{55}}{8}$



$1^2=\left(\frac{3}{8}\right)^2+x^2$
$1=\frac{9}{64}+x^2$
$1-\frac{9}{64}=x^2$
$\frac{64}{64}-\frac{9}{64}=x^2$
$x^2=\frac{55}{64}$
$x=-\frac{\sqrt{55}}{8}$
in Quad. III so must be neg.

6. Which angle, in standard position, shares the same terminal arm with $\frac{-15\pi}{6}$?

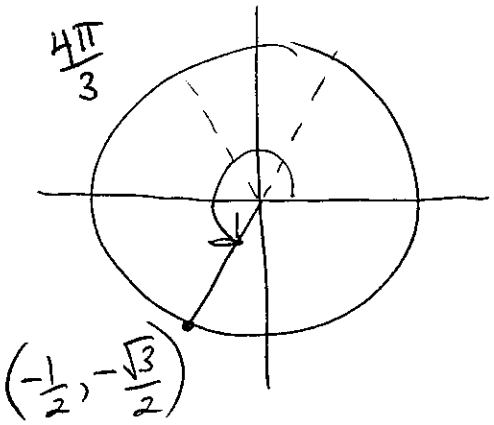
A) $-\frac{2\pi}{3}$
B) $\frac{4\pi}{3}$
C) $\frac{3\pi}{2}$
D) $\frac{5\pi}{3}$



7. For what value of θ is $\tan\theta=\sqrt{3}$ and $\sin\theta=\frac{-\sqrt{3}}{2}$?

A) $\frac{\pi}{3}$
B) $\frac{2\pi}{3}$
C) $\frac{4\pi}{3}$
D) $\frac{5\pi}{3}$

tan is positive & sin is negative
∴ quadrant III



$\tan\theta=\frac{-\frac{\sqrt{3}}{2}}{\frac{-1}{2}}=-\frac{\sqrt{3}}{2}\cdot\frac{2}{1}$
$=\sqrt{3}$.

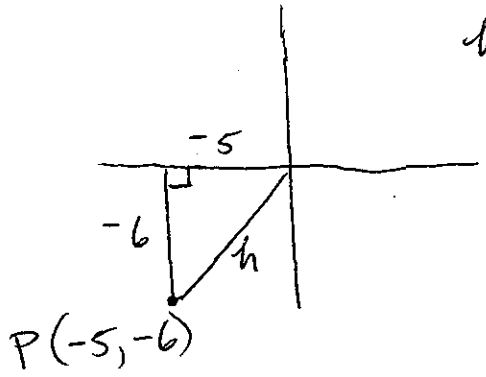
8. Given $P(-5,-6)$ are the coordinates on the terminal arm of an angle θ in standard position, what is $\csc\theta$?

A) $\frac{5}{6}$

B) $\frac{6}{5}$

C) $\frac{\sqrt{61}}{-6}$

D) $\frac{\sqrt{61}}{-5}$



$h^2 = 5^2 + 6^2$

$h^2 = 25 + 36$

$h^2 = 61$

$h = \sqrt{61}$

$\sin\theta = \frac{-6}{\sqrt{61}}$

$\therefore \csc\theta = \frac{-\sqrt{61}}{6}$

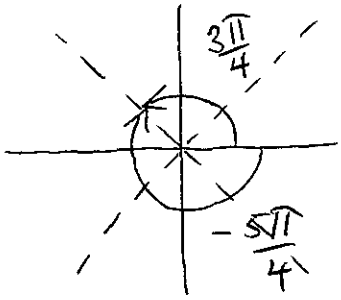
9. Which pair of angles are coterminal?

A) $\frac{5\pi}{4}$ *and* $\frac{-5\pi}{4}$

B) $\frac{3\pi}{4}$ *and* $\frac{-5\pi}{4}$

C) $\frac{-5\pi}{4}$ *and* $\frac{-10\pi}{4}$

D) $\frac{5\pi}{4}$ *and* $\frac{15\pi}{4}$



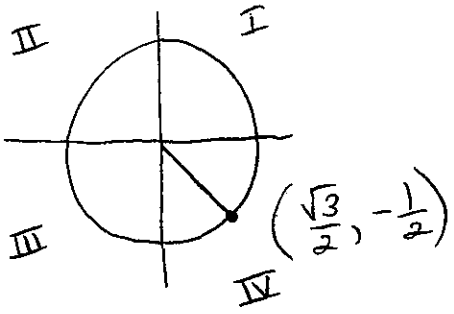
10. A rotation of θ gives a terminal arm in quadrant IV and $\cos\theta = \frac{\sqrt{3}}{2}$. What is $\cot\theta$?

A) $\frac{-1}{\sqrt{3}}$

B) $\frac{1}{\sqrt{3}}$

C) $\sqrt{3}$

D) $-\sqrt{3}$



$\cot\theta = \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}} = \frac{\sqrt{3}}{2} \div \frac{2}{1}$

$= -\sqrt{3}$

11. Solve for x: $\csc^2\theta = 4$, where $0° \le x \le 360°$.

A) $\{30°,150°,210°,330°\}$

B) $\{45°,135°,225°,315°\}$

C) $\{60°,120°\}$

D) $\{240°,300°\}$

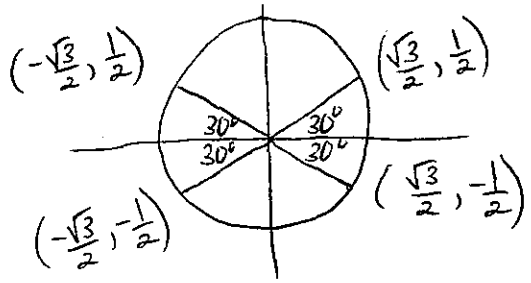
$\csc^2\theta = 4$

$\frac{1}{\sin^2\theta} = 4$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\sqrt{\frac{1}{4}}$

$\sin\theta = \pm\frac{1}{2}$



12. Solve for θ: $\cot\theta - 1 = 0$, where $-2\pi \le x \le 2\pi$.

A) $\left\{\frac{-3\pi}{4}, \frac{-7\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}\right\}$

B) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}, \frac{-\pi}{4}, \frac{-5\pi}{4}\right\}$

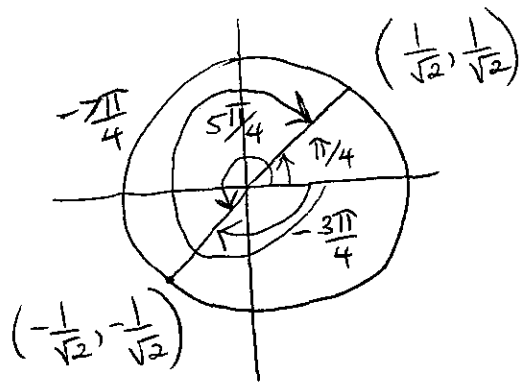
C) $\left\{\frac{3\pi}{4}, \frac{-7\pi}{4}, \frac{\pi}{4}, \frac{-5\pi}{4}\right\}$

D) $\left\{\frac{-3\pi}{4}, \frac{7\pi}{4}, \frac{-\pi}{4}, \frac{5\pi}{4}\right\}$

$\cot\theta - 1 = 0$

$\cot\theta = 1$

$\therefore \tan\theta = 1$



$\frac{\pi}{4}, \frac{5\pi}{4}, -\frac{3\pi}{4}, -\frac{7\pi}{4}$

13. In what step does the first error occur in the solution:

$2\cos^2\theta - \cos\theta - 1 = 0,\ 0° \le \theta < 360°$

STEP 1: $(2\cos\theta + 1)(\cos\theta - 1) = 0$ ✓ ~ $2\cos\theta + 1 = 0$ $\quad \cos\theta - 1 = 0$

STEP 2: $\cos\theta = \frac{-1}{2}$ ✓, $\cos\theta = -1$ ✗ $\quad 2\cos\theta = -1$ $\quad \cos\theta = 1$

$\cos\theta = -\frac{1}{2}$

STEP 3: $\theta = \cos^{-1}\left(\frac{-1}{2}\right), \theta = \cos^{-1}(-1)$

STEP 4: $\theta = 120°, 270°$

A) Step 1
B) Step 2
C) Step 3
D) Step 4

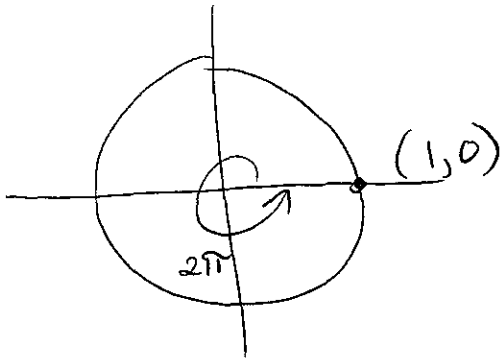
14. How many solutions exist for the equation $\sec\theta = 1$ within the restricted domain $\theta \in (-2\pi, 2\pi]$? ~ same as $-2\pi < \theta \le 2\pi$

A) 1
B) 2
C) 3
D) 4

$\sec\theta = 1$

$\frac{1}{\cos\theta} = 1$

$\cos\theta = 1$



$\therefore \{0, 2\pi\}$.

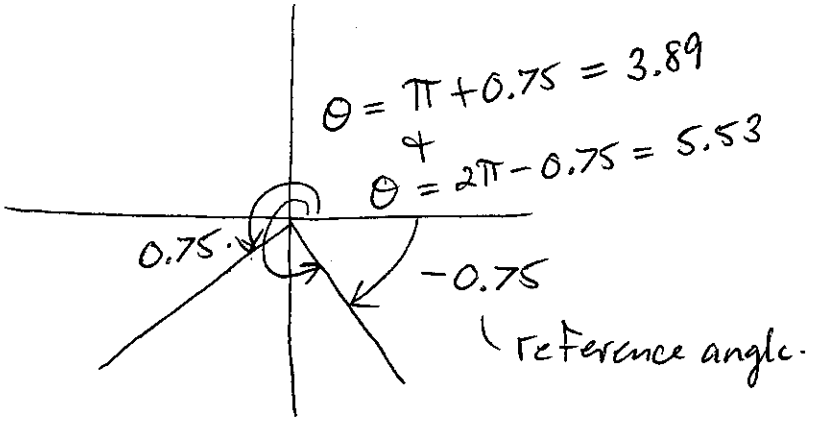
15. Solve $\csc\theta = -1.468,\ 0 \le \theta \le 2\pi$. ~ note restriction ... only pos. answers.

A) $\{-0.78, -2.36\}$

B) $\{-0.75, -2.39\}$

C) $\{3.92, 5.50\}$

D) $\{3.89, 5.53\}$



$\csc\theta = -1.468$

$\frac{1}{\sin\theta} = -1.468$

$\sin\theta = \frac{1}{-1.468}$

$\sin\theta = -0.6812$

$\theta = \sin^{-1}(-0.6812) = -0.75$

(calculator in RAD mode.

Name: _____

Part I: ANSWER SHEET

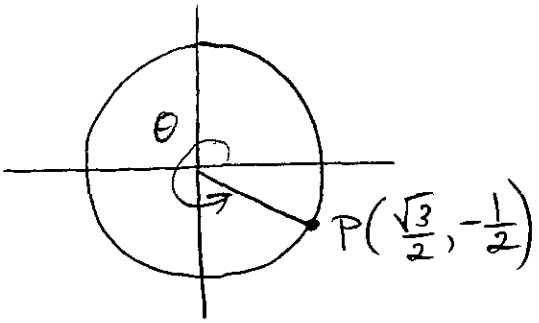
1. B 2. C 3. B 4. C 5. D
6. C 7. C 8. C 9. B 10. D
11. A 12. A 13. B 14. B 15. D.

Part II: CONSTRUCTED RESPONSE

Complete each item in the space provided. Be sure to read each question carefully and provide all necessary details as part of your solution. (15 marks)

1. Given that the point $P\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$ lies at the intersection of the unit circle and the terminal arm of an angle in standard position.

A) Sketch the diagram (1 mark)

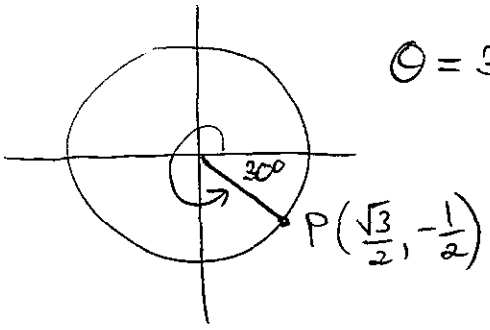


B) Determine the values of the six trigonometric ratios (3 marks)

$\sin\theta = \boxed{-\frac{1}{2}}$ $\tan\theta = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$

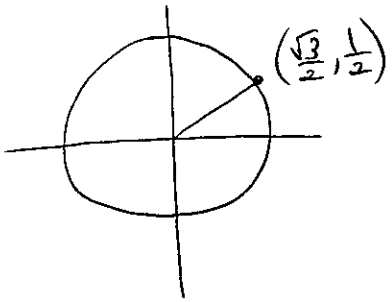
$\cos\theta = \boxed{\frac{\sqrt{3}}{2}}$ $\csc\theta = \boxed{-2}$ $\sec\theta = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$ $\cot\theta = \boxed{-\sqrt{3}}$

C) Determine the angle of rotation in standard position (1 mark)

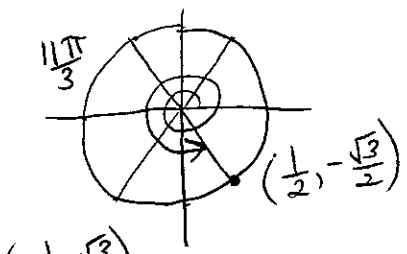


$\theta = 360° - 30° = 330°$

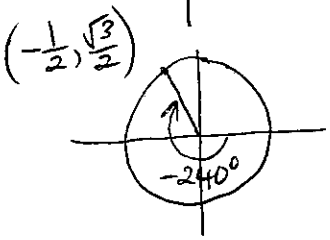
2. Find the exact value for the following expression: $\dfrac{\cot\left(\frac{\pi}{6}\right)+\cos\left(\frac{11\pi}{3}\right)}{\csc(-240°)}$



$\cot\left(\frac{\pi}{6}\right)=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\frac{\sqrt{3}}{2}\cdot\frac{2}{1}=\boxed{\sqrt{3}}$



$\cos\left(\frac{11\pi}{3}\right)=\boxed{\frac{1}{2}}$



$\csc(-240°)=\dfrac{1}{\sin(-240°)}$

$=\dfrac{1}{\frac{\sqrt{3}}{2}}=\dfrac{2}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\boxed{\dfrac{2\sqrt{3}}{3}}$

$=\dfrac{\frac{2\sqrt{3}}{2}+\frac{1}{2}}{\frac{2\sqrt{3}}{3}}=\dfrac{\frac{2\sqrt{3}+1}{2}}{\frac{2\sqrt{3}}{3}}$

$=\dfrac{2\sqrt{3}+1}{2}\cdot\dfrac{3}{2\sqrt{3}}=\dfrac{6\sqrt{3}+3}{4\sqrt{3}}$

$=\dfrac{6\sqrt{3}+3}{4\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{6(3)+3\sqrt{3}}{4(3)}$

$=\dfrac{18+3\sqrt{3}}{12}$

$=\boxed{\dfrac{3}{2}+\dfrac{\sqrt{3}}{4}}$

3. Solve the trigonometric equation where $x\in[-2\pi,2\pi]$:

$2\cos^2 x+5\cos x-3=0$

$(2\cos x-1)(\cos x+3)=0$

$2\cos x-1=0$

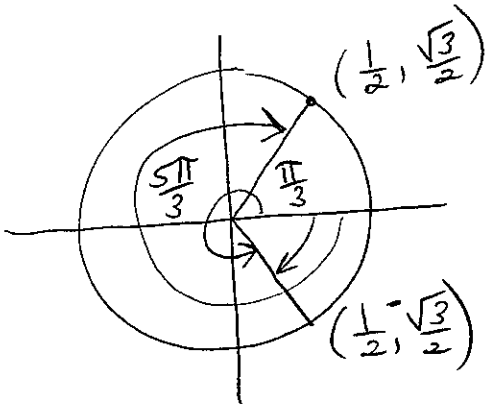
$2\cos x=1$

$\cos x=\frac{1}{2}$

$\cos x+3=0$

$\cos x=-3$

impossible!



$x=\left\{\frac{\pi}{3},\frac{5\pi}{3},-\frac{\pi}{3},-\frac{5\pi}{3}\right\}$.