Name: ANSWER KEY SUGGESTIONS

Teacher: HP

DO NOT OPEN THE EXAMINATION PAPER UNTIL
YOU ARE TOLD BY THE SUPERVISOR TO BEGIN

Newfoundland & Labrador English School District

Mathematics 2200

COMMON FINAL EXAM
June 2014

Value: 70 Marks
Duration: 2 Hours

General Instructions

Part I – Selected Response (35 marks)
Answer ALL questions in this section. Select the letter of the correct response from those provided and shade the letter on the computer scorable card and/or place the letter in the blank provided on the Selected Response Answer Sheet.

Part II - Constructed Response (35 marks)
Answer ALL questions completely and show workings in the space provided.

NOTE:
- **DIAGRAMS ARE NOT NECESSARILY DRAWN TO SCALE**
- **FORMULA SHEET IS PROVIDED**

Student Checklist

- ✓ Write your name and your teacher's name on this exam and on the Part I answer sheet.
- ✓ Check the exam to ensure that there are no missing pages.

Part I - Total Value: 35 marks

Shade the letter of the correct response on the computer scorable card and/or place the letter in the blank provided on the Selected Response Answer Sheet.

1. If -69 is the 17th term in the sequence $\{\ldots,-54,-59,-64,-69,\ldots\}$, what is the first term?

   (A) -154

   (B) -149

   ✓(C) 11

   (D) 16

2. Which best describes a sequence that is geometric and converging?

   (A) $t_n = \frac{2^n}{3}$

   ✓ (B) $t_n = \left(\frac{2}{3}\right)^n$

   (C) $t_n = \left(\frac{3}{2}\right)^n$

   (D) $t_n = 2^n$

3. The sum of an infinite geometric series is 40. If the first term is 10, what is the common ratio?

   (A) $-\frac{5}{4}$

   (B) $-\frac{3}{4}$

   ✓ (C) $\frac{3}{4}$

   (D) $\frac{5}{4}$

4. What is the seventh term in the arithmetic sequence $\{2a\ ,\ a+b\ ,\ 2b,\ldots\}$?

   ✓ (A) $-4a+6b$

   (B) $-4a-6b$

   (C) $8a-6b$

   (D) $8a+6b$

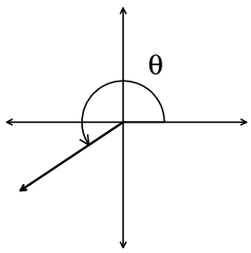
5. What is the measure of the reference angle, in degrees, for $\theta = 217^\circ$ in the graph below?

✓ (A) 37

(B) 53

(C) 143

(D) 217



6. If $P(5,-8)$ is on the terminal arm of θ, what is the measure of θ, to the nearest degree?

(A) 32

(B) 58

✓ (C) 302

(D) 328

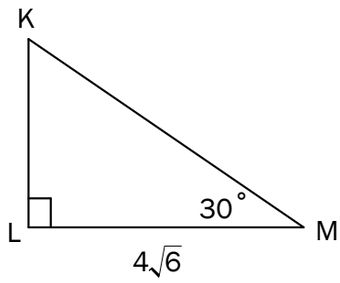
7. If $\angle L = 90^\circ$, $\angle M = 30^\circ$, and $LM = 4\sqrt{6}$ in ΔKLM, what is the exact length of KL?

(A) $2\sqrt{6}$

✓ (B) $4\sqrt{2}$

(C) $12\sqrt{2}$

(D) $8\sqrt{6}$



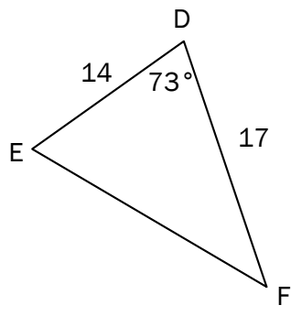
8. If $DE = 14$, $DF = 17$, and $\angle D = 73^\circ$ in ΔDEF, what is the measure of EF, to the nearest tenth?

✓ (A) 18.6

(B) 20.4

(C) 23.5

(D) 25.0



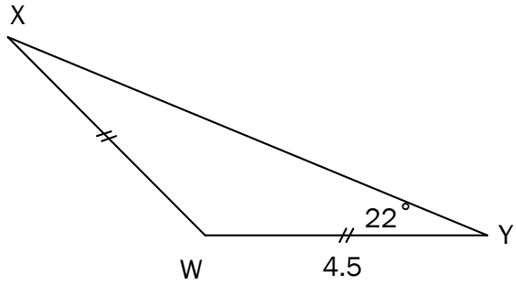
9. If $\angle Y = 22^\circ$, $WY = 4.5$, and $WY = WX$, what is the length of $\overline{XY}$ in ΔWXY?

(A) 2.4

(B) 4.5

✓ (C) 8.3

(D) 11.1



10. Which quadratic function has the widest graph?

✓ (A) $y = \frac{1}{6}(x+2)^2 - 3$

(B) $y = \frac{1}{3}x^2 + 5x + 6$

(C) $y = 2(x-1)^2 + 4$

(D) $y = 4x^2 - 3x - 1$

11. What is the range of the function $y = -(x-1)^2 + 7$?

(A) $\{y \mid y \leq -7,\ y \in R\}$

(B) $\{y \mid y \geq -7,\ y \in R\}$

✓ (C) $\{y \mid y \leq 7,\ y \in R\}$

(D) $\{y \mid y \geq 7,\ y \in R\}$

12. What is the y-intercept of the graph of $y = 2x^2 - 4x$?

(A) -4

(B) -2

✓ (C) 0

(D) 1

13. What is the value of ***a*** in the function $y = ax^2 - 48x + 12$, if the axis of symmetry for the graph of the function is $x = 6$?

(A) -4

(B) -1

(C) 1

✓ (D) 4

14. What is the equation of the function $y = -2x^2 + 8x - 1$ written in the form $y = a(x-p)^2 + q$?

(A) $y = -2(x-2)^2 - 5$

✓ (B) $y = -2(x-2)^2 + 7$

(C) $y = -2(x+2)^2 - 9$

(D) $y = -2(x+2)^2 + 3$

15. A theatre seats 2000 people and when it charges $10 per ticket, all tickets are sold. A survey indicates that for every $2 increase in ticket price, the number of tickets sold would decrease by 100. What equation could be used to determine the ticket price that results in the greatest revenue? Note: n = # of $2 increases

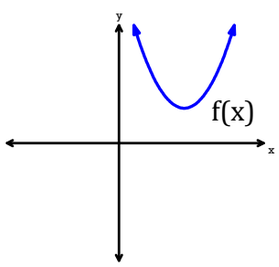
(A) $y=(10-2n)(2000+100n)$

✓ (B) $y=(10+2n)(2000-100n)$

(C) $y=(10-n)(2000+100n)$

(D) $y=(10+n)(2000-100n)$

16. The graph of a quadratic function is shown. What is a possible value for the discriminant of the related equation $f(x)=0$?



✓ (A) -1

(B) 0

(C) $\sqrt{2}$

(D) $\frac{3}{2}$

17. What are the zeroes of the function $f(x)=2x^2-x-3$?

(A) $\left\{-\frac{3}{2}, 1\right\}$

(B) $\left\{-\frac{2}{3}, 1\right\}$

(C) $\left\{\frac{2}{3}, -1\right\}$

✓ (D) $\left\{\frac{3}{2}, -1\right\}$

18. Which value for ***b*** will make the polynomial $x^2+bx+12$ a perfect square trinomial?

(A) $2\sqrt{3}$

(B) 6

✓ (C) $4\sqrt{3}$

(D) 36

19. Simplify: $-\sqrt[3]{5}-7\sqrt{3}+16\sqrt{3}+4\sqrt[3]{5}$

(A) $12\sqrt[3]{3}$

(B) $12\sqrt[3]{8}$

✓ (C) $3\sqrt[3]{5}+9\sqrt{3}$

(D) $3\sqrt[3]{10}+9\sqrt{6}$

20. What is $2xy^2\left(\sqrt[3]{3x}\right)$ written as an entire radical?

(A) $\sqrt[3]{6x^2y^2}$

(B) $\sqrt[3]{12x^3y^4}$

(C) $\sqrt[3]{18x^4y^6}$

✓ (D) $\sqrt[3]{24x^4y^6}$

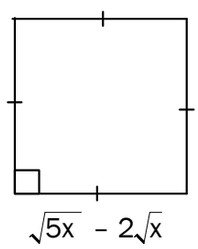
21. What is the area of the square shown with side length $\sqrt{5x}-2\sqrt{x}$?

(A) x

(B) $3x$

(C) $9x-2x\sqrt{10}$

✓ (D) $9x-4x\sqrt{5}$



22. Which rational expression is equivalent to $\frac{4x}{x-4}$, $x \neq 4$?

(A) $\frac{4x^2}{x(x-4)}$

✓ (B) $\frac{-20x}{-5(x-4)}$

(C) $\frac{4x}{(x-4)^2}$

(D) $\frac{x}{x-1}$

23. What are the non-permissible values for $\frac{(w+7)(w-7)}{(w-10)(w+8)} \div \frac{2(w+7)}{w+10}$?

(A) $\{\pm 10, -8\}$

(B) $\{\pm 10, \pm 7, -8\}$

✓ (C) $\{\pm 10, -8, -7\}$

(D) $\{10, -8, -7\}$

24. Simplify completely: $\frac{2x^2}{3-x} - \frac{6x}{3-x}$

✓ (A) $-2x$

(B) $2x$

(C) $\frac{2x^2 - 6x}{3-x}$

(D) $\frac{2x^2 - 6}{3}$

25. Simplify completely: $\frac{a^2 - a - 2}{a+2} \times \frac{a+1}{a^2 - 4}$

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{a^2 + 1}{a^2 + 4}$

✓ (D) $\frac{(a+1)^2}{(a+2)^2}$

26. Which equation could be used to model the solution to the problem below?

It takes Erica 30 minutes less time to shovel the snow off the deck than it takes Kyle. If they work together, they can shovel the deck in 20 minutes. How long would it take Kyle to shovel the deck alone? Note: x = Kyle's time in minutes

(A) $\frac{1}{x-30} + \frac{1}{20} = \frac{1}{x}$

✓ (B) $\frac{1}{x-30} + \frac{1}{x} = \frac{1}{20}$

(C) $\frac{1}{x-20} + \frac{1}{30} = \frac{1}{x}$

(D) $\frac{1}{x-20} + \frac{1}{x} = \frac{1}{30}$

27. Evaluate: $-1|4-8|$

(A) -12

✓ (B) -4

(C) 4

(D) 12

28. If $(-2,-5)$ is on the graph of $y=f(x)$, what is the corresponding point on $y=\left|f(x)\right|$?

(A) $(-2,-5)$

✓ (B) $(-2,5)$

(C) $(2,-5)$

(D) $(2,5)$

29. How many solutions does the equation $\left|x^2-1\right|+5=2$ have?

✓ (A) 0

(B) 1

(C) 2

(D) 3

30. What is(are) the invariant point(s) on $y=x-1$ if its reciprocal, $y=\dfrac{1}{x-1}$, is also graphed?

(A) $\{(-1,-2)\}$

(B) $\{(1,0)\}$

(C) $\{(-1,0),(1,2)\}$

✓ (D) $\{(0,-1),(2,1)\}$

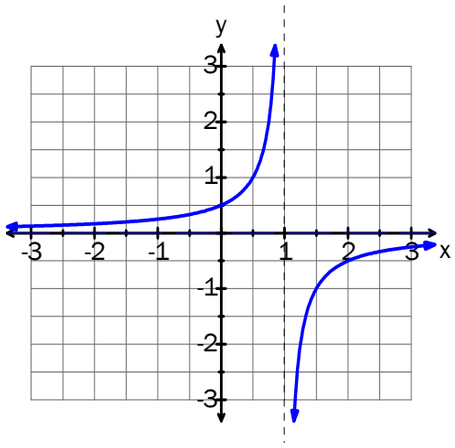
31. Given the graph of $y=\dfrac{1}{f(x)}$, what is the equation of $y=f(x)$?

(A) $y=-2x+1$

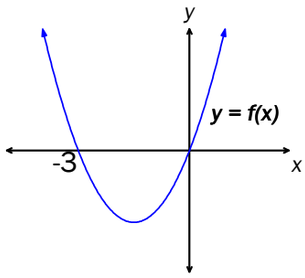
✓ (B) $y=-2x+2$

(C) $y=2x-1$

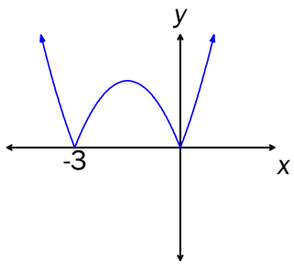
(D) $y=2x-2$



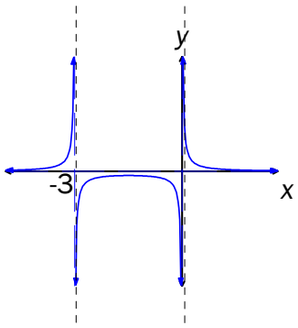
32. Given the graph of $y = f(x)$, which graph is its reciprocal?



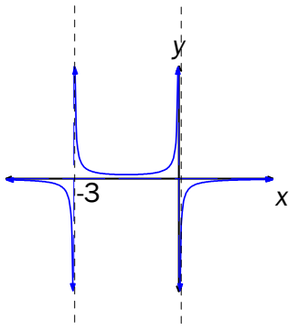
(A)



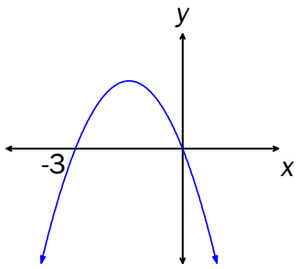
✓ (B)



(C)



(D)



33. What is the solution for the system?

$$\begin{cases} -3x^2 + 5x - y + 6 = 0 \\ 3x^2 + 3x + y + 10 = 0 \end{cases}$$

✓ (A) $(-2, -16)$

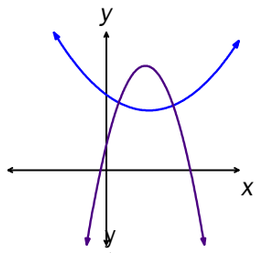
(B) $(-2, 16)$

(C) $(2, -28)$

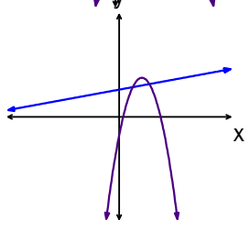
(D) $(2, 28)$

34. Which graph represents the system? $\begin{cases} y = 0.2x^2 - x + 6 \\ y = -1.2x^2 + 5.5x + 2 \end{cases}$

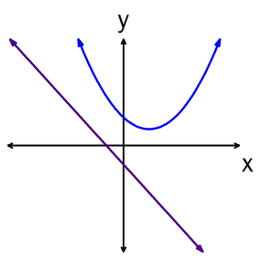
✓ (A)



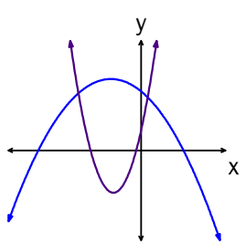
(B)



(C)



(D)



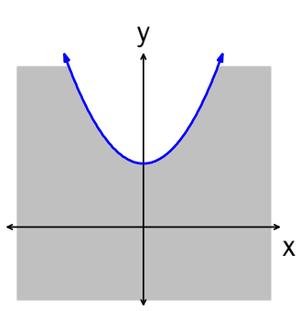
35. Which inequality best represents the graph below?

(A) $y < \frac{1}{2}x^2 + 2$

(B) $y > \frac{1}{2}x^2 + 2$

✓ (C) $y \leq \frac{1}{2}x^2 + 2$

(D) $y \geq \frac{1}{2}x^2 + 2$



Part II - Total Value: 35 marks

Answer ALL questions and show workings in the space provided.

Value

4 36. Find the **exact** sum of the series: $\frac{2}{81}+\frac{2}{27}+\frac{2}{9}+\ \ldots\ +18$

$S_n=\dfrac{t_1(r^n-1)}{r-1}$

$S_n=\dfrac{\frac{2}{81}(3^7-1)}{3-1}$

$S_n=\dfrac{\frac{2}{81}(2186)}{2}$

$S_n=\dfrac{\frac{2}{81}(2186)}{2}$

$S_n=\frac{2}{81}(2186)\left(\frac{1}{2}\right)$

$S_n=\dfrac{2186}{81}$

$\frac{2}{81}\times r=\frac{2}{27}$

$r=\frac{2}{27}\times\frac{81}{2}=3$

$t_n=t_1\,r^{n-1}$

$18=\frac{2}{81}(3)^{n-1}$

$18\times\frac{81}{2}=(3)^{n-1}$

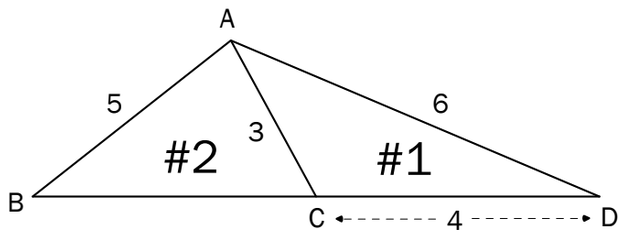
$729=(3)^{n-1}$

$3^6=(3)^{n-1}$

$6=n-1$

$7=n$

4 37. Determine the measure of $\angle B$ to the nearest degree.



$\Delta\#2:\ \angle ACB=180^\circ-117.3^\circ=62.7^\circ$

$\dfrac{\sin 62.7^\circ}{5}=\dfrac{\sin B}{3}$

$\sin B=\dfrac{3\sin 62.7^\circ}{5}$

$B=\sin^{-1}\left(\dfrac{3\sin 62.7^\circ}{5}\right)$

$B=32^\circ$

$\Delta\#1:\ \cos C=\dfrac{d^2+a^2-c^2}{2da}$

$\cos C=\dfrac{3^2+4^2-6^2}{2(3)(4)}$

$\cos C=-\frac{11}{24}$

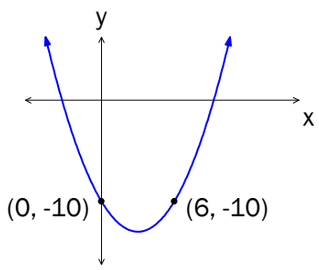
$C=\cos^{-1}\left(-\frac{11}{24}\right)$

$C=117.3^\circ$

$\angle ACD=117.3^\circ$

Value

2 38. Determine the equation, in the form $y = a(x-p)^2 + q$, of the quadratic function that contains the points $(0,-10)$ and $(6,-10)$ and has a minimum value of -13 .



V(3, -13)

$y = a(x-3)^2 - 13$

Sub in (0, -10) $-10 = a(0-3)^2 - 13$

$-10 = 9a - 13$

$-10 + 13 = 9a$

$\frac{3}{9} = \frac{9a}{9}$

$\frac{1}{3} = a$ $\quad \therefore \quad y = \frac{1}{3}(x-3)^2 - 13$

3 39. Algebraically determine the **exact** roots in simplest form for the equation:

$2x(3x-1) = 5$

$6x^2 - 2x = 5$

$6x^2 - 2x - 5 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(6)(-5)}}{2(6)}$

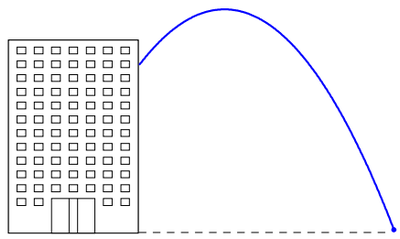
$x = \frac{2 \pm \sqrt{4 + 120}}{12}$

$x = \frac{2 \pm \sqrt{124}}{12}$

$x = \frac{2 \pm \sqrt{4 \times 31}}{12}$ $\quad x = \frac{2 \pm 2\sqrt{31}}{12}$ $\quad x = \frac{1 \pm \sqrt{31}}{6}$

Value

3 40. A ball is thrown from a balcony on a building and its path is represented by the function $y = -5x^2 + 20x + 60$, where x is the distance (in m) from the building along the ground, and y is the height (in m) above the ground. If the parking lot extends 5 m from the building, will the ball land on the parking lot or beyond? Justify your answer.



$$0 = -5x^2 + 20x + 60$$

$$0 = -5(x^2 - 4x - 12)$$

$$\frac{0}{-5} = \frac{-5}{-5}(x^2 - 4x - 12)$$

$$0 = x^2 - 4x - 12$$

$$0 = (x - 6)(x + 2)$$

$x = 6$ *or* $x = -2$

reject

Since the ball lands 6 m from the building and the parking lot only extends 5 m, the ball will land beyond the parking lot.

2 41. Rationalize the denominator and simplify: $\dfrac{\sqrt{6}}{4 - \sqrt{2}}$

$$\frac{\sqrt{6}}{4 - \sqrt{2}} \times \frac{4 + \sqrt{2}}{4 + \sqrt{2}}$$

$$= \frac{4\sqrt{6} + \sqrt{12}}{16 + 4\sqrt{2} - 4\sqrt{2} - 2}$$

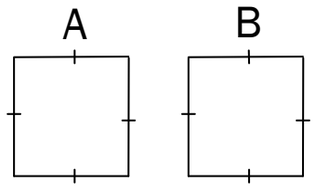
$$= \frac{4\sqrt{6} + \sqrt{4 \times 3}}{14}$$

$$= \frac{4\sqrt{6} + 2\sqrt{3}}{14}$$

$$= \frac{2\sqrt{6} + \sqrt{3}}{7}$$

Value

3 42. The areas of congruent squares A and B are represented by $\sqrt{3x+1}$ square units and $(x-1)$ square units, respectively. Algebraically determine the area of each square.



Area = $\sqrt{3x+1}$

Area = $(x-1)$

$$\sqrt{3x+1} = x-1$$
$$\left(\sqrt{3x+1}\right)^2 = (x-1)^2$$
$$3x+1 = (x-1)(x-1)$$
$$3x+1 = x^2-2x+1$$
$$0 = x^2-2x+1-3x-1$$
$$0 = x^2-5x$$
$$0 = x(x-5)$$
$$x = 0 \quad or \quad x = 5$$

reject

Check x = 0

$LS = \sqrt{3(0)+1} = 1$ $\qquad RS = 0-1 = -1$

not valid ∴ reject

Check x = 5

$LS = \sqrt{3(5)+1} = 4$ $\qquad RS = 5-1 = 4$

valid

$\therefore$ Area $= (5-1) = 4$ square units

3 43. Solve algebraically: $\dfrac{n+9}{n+2} - 1 = \dfrac{6}{n+1}$

$$(n+2)(n+1)\frac{(n+9)}{(n+2)} - 1(n+2)(n+1) = (n+2)(n+1)\frac{6}{(n+1)}$$

$$(n+1)(n+9) - (n+2)(n+1) = 6(n+2)$$

$$(n^2+10n+9) - (n^2+3n+2) = 6n+12$$

$$n^2+10n+9-n^2-3n-2-6n-12 = 0$$

$$n-5 = 0$$

$$n = 5$$

Check x = 5

$LS = \dfrac{5+9}{5+2} - 1 = \dfrac{14}{7} - 1 = 1$ $\qquad RS = \dfrac{6}{5+1} = \dfrac{6}{6} = 1$

valid

OR

$$\frac{n+9}{n+2} - \frac{n+2}{n+2} = \frac{6}{n+1}$$

$$\frac{n+9-n-2}{n+2} = \frac{6}{n+1}$$

$$\frac{7}{n+2} = \frac{6}{n+1}$$

$$7(n+1) = 6(n+2)$$
$$7n+7 = 6n+12$$
$$7n-6n = 12-7$$
$$n = 5$$

Value

4 44. Solve algebraically: $\left|x^2-6x\right|=2x-12$

$+(x^2-6x)=2x-12$
$x^2-6x-2x+12=0$
$x^2-8x+12=0$
$(x-6)(x-2)=0$
$x=6 \quad x=2$

$-(x^2-6x)=2x-12$
$x^2-6x=-2x+12$
$x^2-6x+2x-12=0$
$x^2-4x-12=0$
$(x-6)(x+2)=0$
$x=6 \quad x=-2$

Check x = 6

$LS=\left|6^2-6(6)\right| \quad RS=2(6)-12$
$=\left|0\right| \quad =12-12$
$=0 \quad =0$
valid

Check x = 2

$LS=\left|2^2-6(2)\right| \quad RS=2(2)-12$
$=\left|4-12\right| \quad =4-12$
$=8 \quad =-8$
not valid $\therefore$ reject

Check x = – 2

$LS=\left|(-2)^2-6(-2)\right| \quad RS=2(-2)-12$
$=\left|4+12\right| \quad =-4-12$
$=16 \quad =-16$
not valid $\therefore$ reject

$\therefore$ x = 6

3 45. Algebraically determine the solution(s) to the system:

$$\begin{cases} y=2x-10 \\ y=x^2-6x+5 \end{cases}$$

$2x-10=x^2-6x+5$

$0=x^2-6x+5-2x+10$

$0=x^2-8x+15$
$0=(x-5)(x-3)$
$x=5 \quad x=3$

if $x=5$
$y=2(5)-10=10-10=0$

if $x=3$
$y=2(3)-10=6-10=-4$

Solutions: $(5,0)$ and $(3,-4)$

Value

4 46. A rectangular piece of land is to be enclosed by 140 m of fencing. Write, and algebraically solve, a quadratic inequality to determine the possible dimensions that will allow the area of the enclosed land to be greater than 1200 m^2.

Area > 1200

$x(70-x) > 1200$

$70x - x^2 > 1200$

$-x^2 + 70x - 1200 > 0$

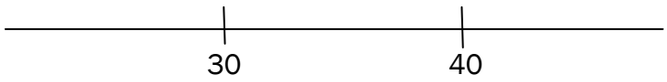
$x^2 - 70x + 1200 < 0$

$(x-30)(x-40) < 0$

$x = 30 \quad x = 40$

x = width

70 - x = length



Test 20	Test 35	Test 50
$70(20) - 20^2$	$70(35) - 35^2$	$70(50) - 50^2$
= 1000	= 1225	= 1000
1000 < 1200	1225 > 1200	1000 < 1200
∴ No	∴ Yes	∴ No

30 < x < 40 ∴ The width must be between 30 m and 40 m.