

The Discriminant

$$b^2 - 4ac$$

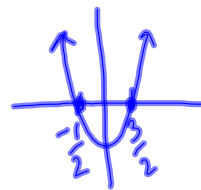
Three possible outcomes for our roots / x-intercepts.

$$1) \frac{2 \pm \sqrt{16}}{4} \quad \boxed{b^2 - 4ac > 0}$$

⇒ Two different real roots

$$\frac{2 \pm 4}{4} \rightarrow \frac{2+4}{4} = \frac{6}{4} = \frac{3}{2}$$

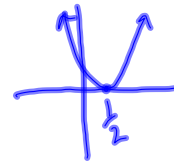
$$\frac{2-4}{4} = \frac{-2}{4} = -\frac{1}{2}$$



$$2) \boxed{b^2 - 4ac = 0}$$

$$\text{ex) } \frac{2 \pm \sqrt{0}}{4} \rightarrow \frac{2+0}{4} \Rightarrow \frac{1}{2}$$

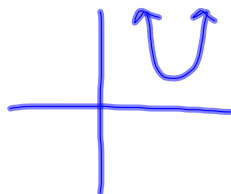
$$\frac{2-0}{4} \Rightarrow \frac{1}{2}$$



⇒ One real root (double root)
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 Two equal real roots

$$3) \boxed{b^2 - 4ac < 0}$$

$$\text{ex) } \frac{2 \pm \sqrt{-16}}{4} \leftarrow \begin{array}{l} \text{imaginary roots} \\ \text{or} \\ \text{No real roots} \end{array}$$



Summary

The discriminant is used to determine the Nature of the roots.

- 1) $b^2 - 4ac > 0 \Rightarrow$ Two unequal real roots
 \Rightarrow crosses x-axis twice
- 2) $b^2 - 4ac = 0 \Rightarrow$ Two equal real roots
 \Rightarrow one x-intercept
- 3) $b^2 - 4ac < 0 \Rightarrow$ Two unequal imaginary roots
 \Rightarrow Does not cross x-axis

Examples :

- 1) Determine the nature of the roots

$$3x^2 - 2x + 1 = 0$$

$$b^2 - 4ac$$

$$(-2)^2 - 4(3)(1)$$

$$4 - 12$$

$$-8$$

$$D = -8$$

$$D < 0 \Rightarrow \text{No x-intercepts}$$

- 2) Which one of the following could have a discriminant of 0?

- a) $y = 2(x-1)^2 + 6$

- b) $y = -2(x-1)^2 + 6$

- c) $y = -2(x-1)^2$**

- d) $y = x^2 + 6$

