

7. Solve for θ in the specified domain. Give solutions as exact values where possible. Otherwise, give approximate measures to the nearest thousandth.

- a) $2 \cos^2 \theta - 3 \cos \theta + 1 = 0, 0 \leq \theta < 2\pi$
 b) $\tan^2 \theta - \tan \theta - 2 = 0, 0^\circ \leq \theta < 360^\circ$
 c) $\sin^2 \theta - \sin \theta = 0, \theta \in [0, 2\pi)$
 d) $\sec^2 \theta - 2 \sec \theta - 3 = 0,$
 $\theta \in [-180^\circ, 180^\circ)$

Apply

9. Aslan and Shelley are finding the solution for $2 \sin^2 \theta = \sin \theta, 0 < \theta \leq \pi$. Here is their work.

$$\begin{aligned} 2\sin^2 \theta &= \sin \theta \\ \frac{2\sin^2 \theta}{\sin \theta} &= \frac{\sin \theta}{\sin \theta} && \text{Step 1} \\ 2\sin \theta &= 1 && \text{Step 2} \\ \sin \theta &= \frac{1}{2} && \text{Step 3} \\ \theta &= \frac{\pi}{6}, \frac{5\pi}{6} && \text{Step 4} \end{aligned}$$

- a) Identify the error that Aslan and Shelley made and explain why their solution is incorrect.
 b) Show a correct method to determine the solution for $2 \sin^2 \theta = \sin \theta, 0 < \theta \leq \pi$.

13. a) Helene is asked to solve the equation $3 \sin^2 \theta - 2 \sin \theta = 0, 0 \leq \theta \leq \pi$. She finds that $\theta = \pi$. Show how she could check whether this is a correct root for the equation.
 b) Find all the roots of the equation $3 \sin^2 \theta - 2 \sin \theta = 0, \theta \in [0, \pi]$.

16. Nora is required to solve the following trigonometric equation.

$$9 \sin^2 \theta + 12 \sin \theta + 4 = 0, \theta \in [0^\circ, 360^\circ)$$

Nora did the work shown below. Examine her work carefully. Identify any errors.

Rewrite the solution, making any changes necessary for it to be correct.

$$9 \sin^2 \theta + 12 \sin \theta + 4 = 0$$

$$(3 \sin \theta + 2)^2 = 0$$

$$3 \sin \theta + 2 = 0$$

Therefore, $\sin \theta = -\frac{2}{3}$

Use a calculator.

$$\sin^{-1}\left(-\frac{2}{3}\right) = -41.8103149$$

So, the reference angle is 41.8° , to the nearest tenth of a degree.

Sine is negative in quadrants II and III.

The solution in quadrant II is $180^\circ - 41.8^\circ = 138.2^\circ$.

The solution in quadrant III is $180^\circ + 41.8^\circ = 221.8^\circ$.

Therefore, $\theta = 138.2^\circ$ and $\theta = 221.8^\circ$, to the nearest tenth of a degree.

7. a) $\theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}$
 b) $\theta = 63.435^\circ, 243.435^\circ, 135^\circ, 315^\circ$
 c) $\theta = 0, \frac{\pi}{2}, \pi$
 d) $\theta = -180^\circ, -70.529^\circ, 70.529^\circ$

9. a) They should not have divided both sides of the equation by $\sin \theta$. This will eliminate one of the possible solutions.

b)

$$2 \sin^2 \theta = \sin \theta$$

$$2 \sin^2 \theta - \sin \theta = 0$$

$$\sin \theta (2 \sin \theta - 1) = 0$$

$$\sin \theta = 0 \quad \text{and} \quad 2 \sin \theta - 1 = 0$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \pi$$