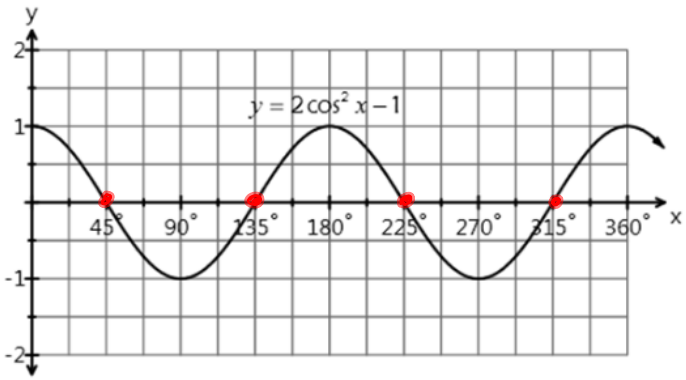
5.4 Equations and Graphs of Trigonometric Functions

You can represent phenomena with periodic behaviour or wave characteristics by trigonometric functions or model them approximately with sinusoidal functions. You can identify a trend or pattern, determine an appropriate mathematical model to describe the process, and use it to make predictions (interpolate or extrapolate).

Example 1

Solve a Trigonometric Equation in Degrees

Determine the solutions for the trigonometric equation $2\cos^2 x - 1 = 0$ for the interval $0° \le x \le 360°$.



Solution

Method 1: Solve Graphically

The solutions to the equation $2\cos^2 x - 1 = 0$ for $0° \le x \le 360°$ are the x-intercepts of the graph of the related function. Thus the solutions are:

$x = 45°, 135°, 225°, 315°$

Method 2: Solve Algebraically

$2\cos^2 x - 1 = 0$

$2\cos^2 x = 1$

$\cos^2 x = \frac{1}{2}$

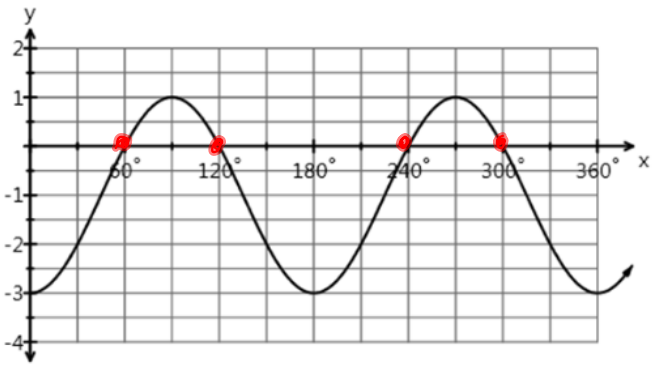
$\cos x = \pm\sqrt{\frac{1}{2}}$

$\cos x = \pm\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45°$ ref ∠

Your Turn

Determine the solutions for the trigonometric equation $4\sin^2 x - 3 = 0$ for the interval $0° \le x \le 360°$.



check:

$4\sin^2 x = 3$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

Example 2

Solve a Trigonometric Equation in Radians

Determine the general solutions for the trigonometric equation $16 = 6\cos\frac{\pi}{6}x + 14$.
Express your answers to the nearest hundredth.

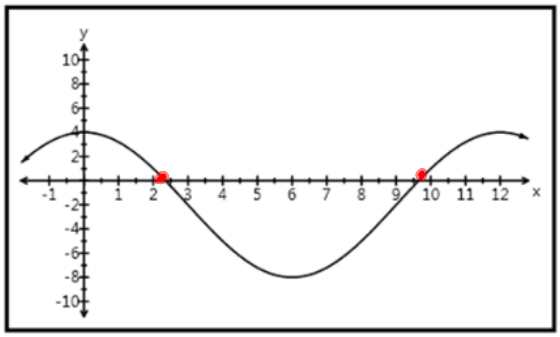
$6\cos\frac{\pi}{6}x - 2 = 0$

Solution

Method 1: Determine the Zeros of the Function

Rearrange the equation $16 = 6\cos\frac{\pi}{6}x + 14$ so that one side is equal to 0. Thus consider the graph of:

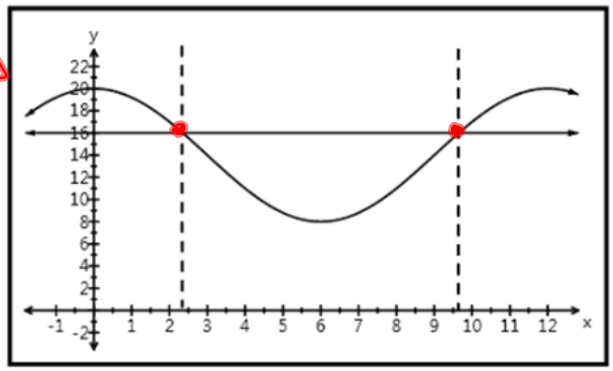
$6\cos\frac{\pi}{6}x - 2 = 0$



The solutions to the equation are the x-intercepts. From looking at the graph the x-intercepts are approximately $x \approx$ 2.4 and $x \approx$ 9.6.

Method 2: Determine the Points of Intersection

Graph the functions $y = 6\cos\frac{\pi}{6}x + 14$ and $y = 16$.



In the interval $0 \le x \le 12$ as shown, the points of intersection are $x \approx$ 2.4 and $x \approx$ 9.6.

The period of the function is 12 radians.

The points of intersection repeat in multiples of 12 radians from each of the intercepts. The general solutions to the original equation are:

$x = \{2.4 + 9.6 + 12 \text{ radians}$

Method 3: Solve Algebraically

Solve for x: $16 = 6\cos\frac{\pi}{6}x + 14$

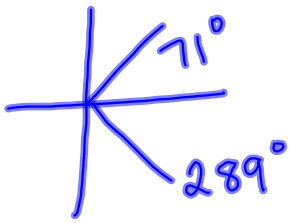
$6\cos\frac{\pi}{6}x = 2$

$\cos\frac{\pi}{6}x = \frac{2}{6}$

$\cos\left(\frac{\pi}{6}x\right) = \frac{1}{3}$

$\cos m = \frac{1}{3}$

$\cos^{-1}\left(\frac{1}{3}\right) \approx 71°$



$\frac{\pi}{6}x = 71°$

$30x = 71$

$x = \frac{71}{30}$

$x \approx 2.4$

$30x = 289$

$x = \frac{289}{30}$

$x \approx 9.6$

1. The partial graphs of the functions $y = 4\sin 2(x + 45°)$ and the line $y = 3$ are shown. Determine the solutions to the equation $4\sin 2(x + 45°) = 3$ over the interval $0° \le x \le 360°$. Express your answers to the nearest degree.