11.2 Combinations

A combination is a selection of a group of objects, taken from a larger group, for which the order in which they are selected is NOT important.

combination

- a selection of objects without regard to order
- all of the three-letter combinations of P, Q, R, and S are PQR, PQS, PRS, and QRS (arrangements such as PQR and RPQ are the same combination)

Example 1a

Calculate the number of combinations of three digits made from the digits 1, 2, 3, 4, and 5 **without** repetitions:

Solution

Number of choices for the first digit		Number of choices for the second digit		Number of choices for the third digit
5	$\cdot$	4	$\cdot$	3

Thus there are $\boxed{5} \times \boxed{4} \times \boxed{3} = \boxed{60}$ ways to arrange 3 items from 5.

However, 3 digits can be arranged in $\boxed{3!}$ ways among themselves.

So, the number of combinations = $\frac{60}{3!} = 10$

The notation ${}_nC_r$, or $\binom{n}{r}$, represents the number of combinations of n items taken r at a time, where $n \geq r$ and $r \geq 0$.

Why must $n \geq r \geq 0$?

$${}_nC_r = \frac{{}_nP_r}{r!}$$

$$= \frac{\frac{n!}{(n-r)!}}{r!}$$

$$= \boxed{\frac{n!}{(n-r)!r!}}$$

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

Example 1

Combinations and the Fundamental Counting Principle

Use combination notation to determine the number of ways of choosing three digits from a group of five digits.

Did You Know?

The number of combinations of n items taken r at a time is equivalent to the number of combinations of n items taken $n - r$ at a time.

${}_nC_r = {}_nC_{n-r}$

Solution

$${}_5C_3 = 10$$

$${}_nC_r = \frac{n!}{(n-r)!\,r!} = \frac{5!}{(5-3)!\,3!}$$

Example 1

Combinations and the Fundamental Counting Principle

There are 12 females and 18 males in a grade 12 class. The principal wishes to meet with a group of 5 students to discuss details about the upcoming graduation.

a) How many selections are possible?

$${}_{30}C_5 = 142\,506$$

b) How many selections are possible if the group consists of two females and three males?

$${}_{12}C_2 \cdot {}_{18}C_3 = 53856$$ possible selections

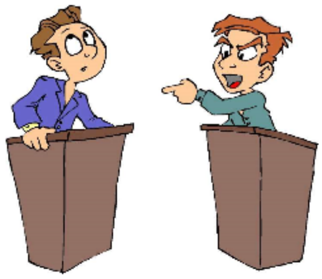
c) One of the female students is named Brooklyn. How many five-member selections consisting of Brooklyn, one other female, and three males are possible?

$${}_{11}C_1 \cdot {}_{18}C_3 = 8976$$

one left to chose from 11 girls

Your Turn

In how many ways can the debating club coach select a team from six grade 11 students and seven grade 12 students if the team has:



a) four members?

Solution

$_{13}C_4 = 715$

b) four members, only one of whom is in grade 11?

Solution

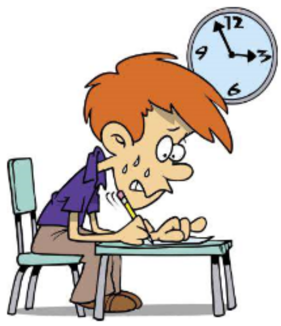
$_6C_1 \cdot {_7C_3} = 210$

grade 11 grade 12

Example 2

Combinations With Cases

Craig is writing a geography exam. The instructions say that he must answer a specified number of questions from each section. How many different selections of questions are possible if:



a) he must answer two of the four questions in part A and three of the five questions in part B?

Solution

$_4C_2 \cdot {_5C_3} = 60$

b) he must answer two of the four questions in part A and at least four of the five questions in part B?

Solution

Case 1: Case 2:

2 from A + 4 in B or 2 from A + 5 in B

$_4C_2 \cdot {_5C_4} + {_4C_2} \cdot {_5C_5}$

$30 + 6$

$= 36$ ways

Your Turn

A bag contains seven black balls and six red balls. In how many ways can you draw groups of five balls if at least three must be red?

7 black 6 red

Choosing 5 balls, at least 3 red

Case 1: 3 Red·2Bl

Case 2: 4 Red·1B

Case 3: 5 Red

$${}_6C_3 \cdot {}_7C_2 + {}_6C_4 \cdot {}_7C_1 + {}_6C_5 = 531 \text{ ways}$$

Example 3

Simplifying Expressions and Solving Equations With Combinations

a) Express as factorials and simplify:

$$\frac{{}_nC_5}{{}_{n-1}C_3}$$

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

Solution

$$\frac{\frac{n!}{(n-5)!\,5!}}{\frac{(n-1)!}{(n-4)!\,3!}}$$

$$\frac{n!}{(n-5)!\,5!} \cdot \frac{(n-4)!\,3!}{(n-1)!}$$

$$= \frac{n \cdot \cancel{(n-1)!}}{\cancel{(n-5)!}\,5!} \cdot \frac{(n-4)\cancel{(n-5)!}\,3!}{\cancel{(n-1)!}}$$

$$\frac{n(n-4)}{5 \cdot 4} = \frac{n^2-4n}{20}$$

b) ~~Solve for n:~~

$$\cancel{2\left({}_nC_2\right) = {}_{n+1}C_3}$$

Solution

b) $2\left({}_nC_2\right) = {}_{n+1}C_3$

$(n+1-3)!$

$$2\left(\frac{n!}{(n-2)!\,2!}\right) = \frac{(n+1)!}{(n-2)!\,3!}$$

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$