

Sec. 1.4: Inverse of a Relation

Inverse of a function: if f is a function with domain A and range B , the inverse will have a domain B and range A .

The inverse will be denoted f^{-1} .

Basically, the x and y -values switch.

$$(x, y) \rightarrow (y, x)$$

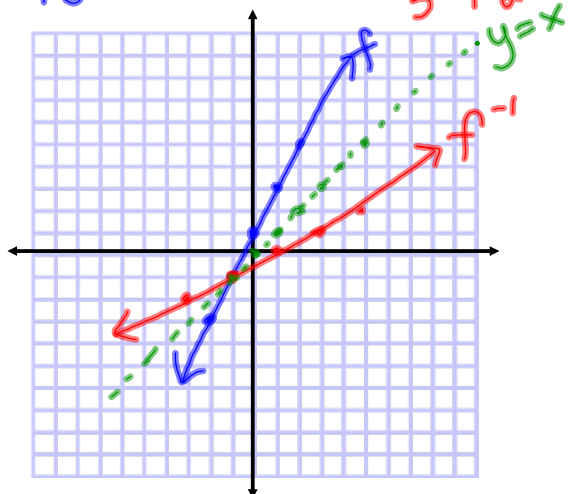
example:

$$f = 2x + 1$$

x	y
-2	-3
-1	-1
0	1
1	3
2	5



x	y
-3	-2
-1	-1
1	0
3	1
5	2

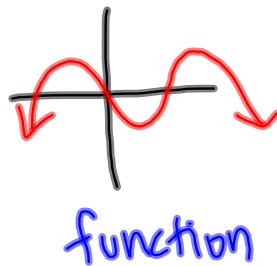
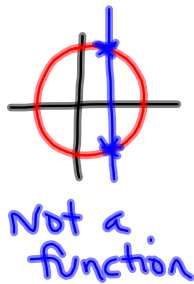
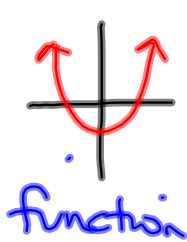


Inverse functions are reflected along the line $y=x$.

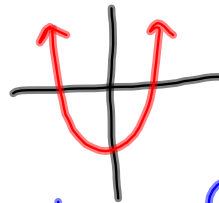
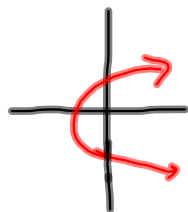
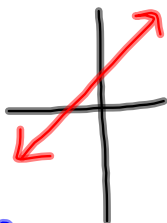
Coordinate points $(-2, -2)$, $(-1, -1)$, $(0, 0)$, $(1, 1)$... will be the same points.

Recall: Vertical Line Test

A test used to determine if a given graph is a function.



For Inverses, we consider The Horizontal Line Test.



f is a function

f is not a function

f is a function
But

f⁻¹ AND

is a function

But

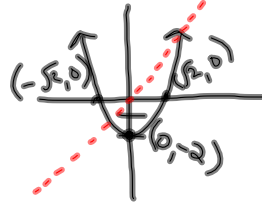
f⁻¹ is a function

f⁻¹ is not a
function

Restricting the Domain on functions to make sure the Inverse will be a function.

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$$f(x) = x^2 - 2$$

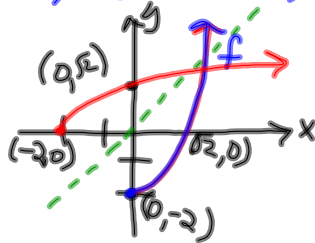


$$\begin{aligned} x^2 - 2 &= 0 \\ x^2 &= 2 \\ x &= \pm\sqrt{2} \end{aligned}$$

$f(x)$ is a function
 $f^{-1}(x)$ is not

If we restricted the domain (cut the parabola in half), the inverse will be a function.

For ex.) $\{x \mid x \geq 0, x \in \mathbb{R}\}$



$$\begin{aligned} &f(x) \\ D: &\{x \mid x \geq 0, x \in \mathbb{R}\} \\ R: &\{y \mid y \geq -2, y \in \mathbb{R}\} \end{aligned}$$

$$\begin{aligned} &f^{-1}(x) \\ D: &\{x \mid x \geq -2, x \in \mathbb{R}\} \\ R: &\{y \mid y \geq 0, y \in \mathbb{R}\} \end{aligned}$$

$$\begin{aligned} f(x) &= x^2 - 2 \\ y &= x^2 - 2 \quad \text{Switch } x \text{ \& } y \\ x &= y^2 - 2 \quad \text{solve for } y \\ x + 2 &= y^2 \\ \pm\sqrt{x+2} &= y \\ \text{one side} &\Rightarrow y = \sqrt{x+2} \end{aligned}$$

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