Section 2.2 : Permutations and Factorial Notation

example: How many ways can 6 students stand in a line?

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$= 720$$

This is called a factorial, written $6!$

Calculator $\Rightarrow 6! = 720$

factorial notation: product of consecutive descending natural numbers

* $0! = 1$

$$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot (n-4) \cdots 3 \cdot 2 \cdot 1$$

ex)

$$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

Evaluate p 9.78

a) $10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

calculator $10! = 3\,628\,800$

b) $\dfrac{12!}{9!\,3!} = 220$

$$\frac{12 \cdot 11 \cdot 10 \cdot \cancel{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{(3 \cdot 2 \cdot 1)(\cancel{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1})}$$

$$\frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220$$

c) $\dfrac{(n+2)!}{(n-1)!} = \dfrac{(n+2)\cdot(n+1)\cdot n \cdot \cancel{(n-1)(n-2)\ldots}}{\cancel{(n-1)(n-2)\ldots}}$

$$= (n+2)(n+1)n$$

d) $\frac{n!}{(n-2)!} = 90$, solve for n.

$$\frac{n\cdot(n-1)\cancel{(n-2)(n-3)(n-4)\cdots 3\cdot 2\cdot 1}}{\cancel{(n-2)(n-3)(n-4)\cdots 3\cdot 2\cdot 1}}$$

$n\cdot(n-1) = 90$

$n^2 - n = 90$

$n^2 - n - 90 = 0$

Solve by factoring or quadratic formula

add → −1
multiply → −90

9 & −10

$(n+9)(n-10)=0$

$n+9=0$ | $n-10=0$

$n=-9$ | $n=10$

Check both possible answers for n

$\frac{n!}{(n-2)!} = 90$ $\qquad$ $\frac{10!}{(10-2)!} = \frac{10!}{8!} = 90$

If $n=-9$

$(-9)!$ error!

Worksheet

1. $0! = 1$

2 a) 30

b) 420

c) 9506

3 a) $\frac{n!}{(n-3)!}$ → 0 or higher
→ 3 or higher

Restriction: $n \geq 3$

b) $\frac{(n+5)!}{(n+2)!}$ → -5 or higher
→ -2 or higher

Restriction: $n \geq -2$